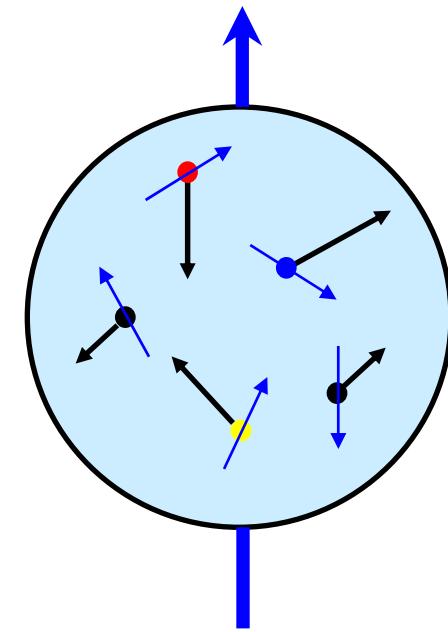


Spin effects and partonic intrinsic \mathbf{k}_\perp

- ❑ Parton intrinsic motion (\mathbf{k}_\perp) in unpolarized inclusive processes
- ❑ \mathbf{k}_\perp and SSA in SIDIS
- ❑ \mathbf{k}_\perp and SSA in pp interactions
- ❑ Spin- \mathbf{k}_\perp correlations in distribution ($f_{q/p}$) and fragmentation ($D_{h/q}$) functions
- ❑ Factorization, QCD evolution, universality with spin- \mathbf{k}_\perp dependent $f_{q/p}$ and $D_{h/q}$



Based on works in collaboration with M. Boglione, U. D'Alesio, A. Kotzinian, E. Leader, S. Melis, F. Murgia and A. Prokudin

Phys. Rev. D71 (2005) 074006; D72 (2005) 094007;
D71 (2005) 014002; Phys. Rev. D73 (2006) 014020

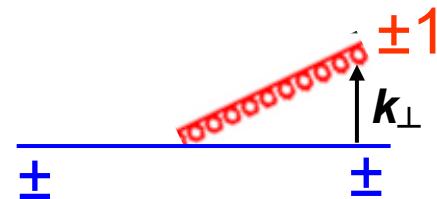
Partonic intrinsic motion

Plenty of theoretical and experimental evidence for transverse motion of partons within nucleons and of hadrons within fragmentation jets

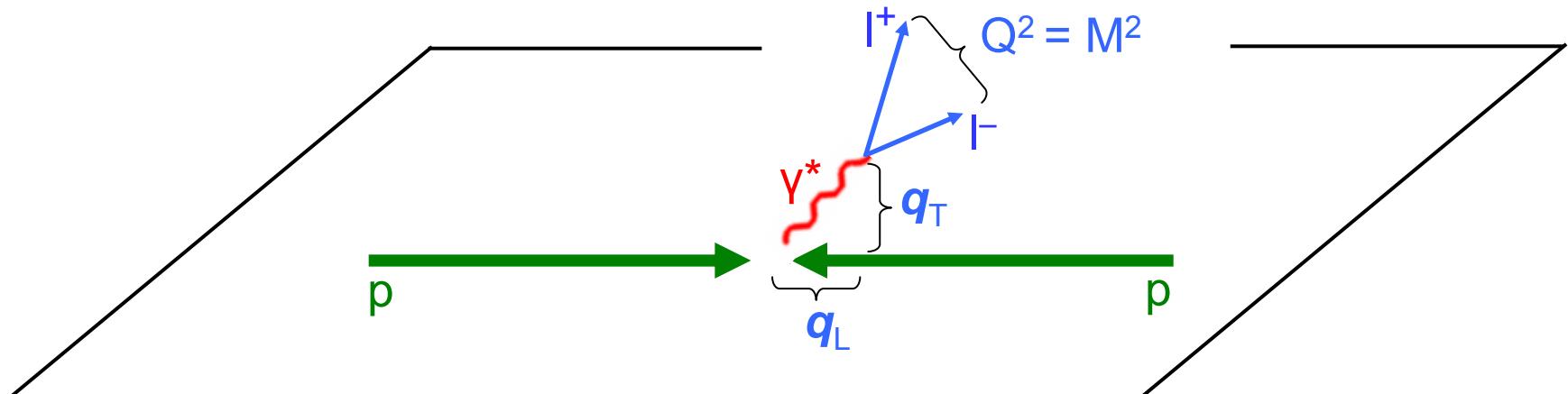
uncertainty principle

$$\Delta x \approx 1 \text{ fm} \Rightarrow \Delta p \approx 0.2 \text{ GeV/c}$$

gluon radiation



q_T distribution of lepton pairs in D-Y processes

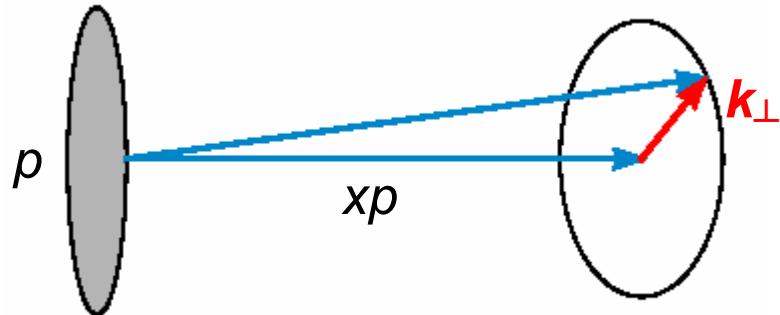


p_T distribution of hadrons in SIDIS

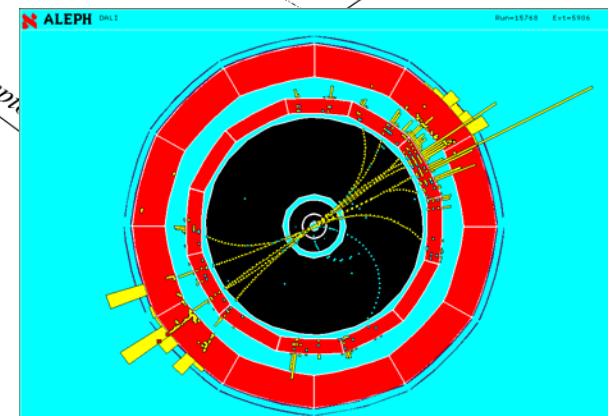
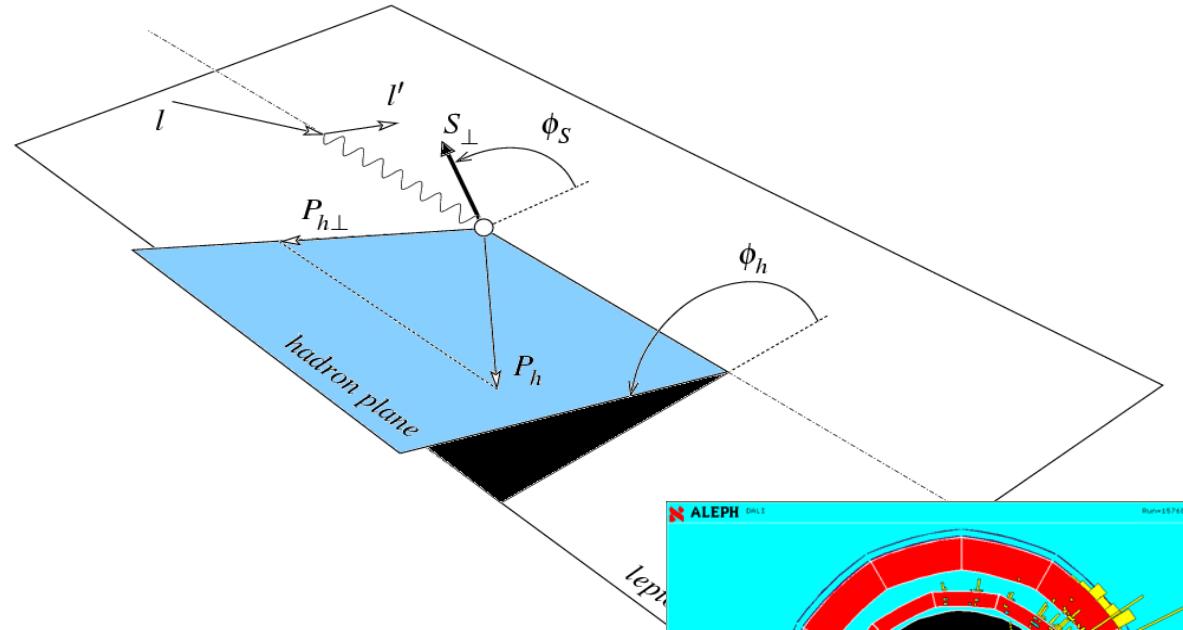
$$\gamma^* p \rightarrow hX$$

Hadron distribution in jets in e^+e^- processes

Large p_T particle production in $pN \rightarrow hX$



Transverse motion is usually integrated, but there might be important spin- k_\perp correlations



Unpolarized SIDIS (LO)

$$d\sigma^{Ip \rightarrow IhX} = \sum_q f_q(x, Q^2) \otimes d\sigma^{Iq \rightarrow Iq} \otimes D_q^h(z, Q^2)$$

In collinear parton model

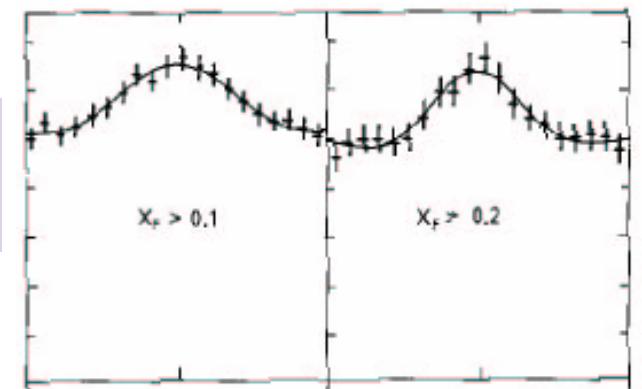
$$x = \frac{Q^2}{2p \cdot q} \quad Q^2 = -q^2 \quad y = \frac{p \cdot q}{l \cdot p}$$

$$\sigma^{Iq \rightarrow Iq} \propto \hat{s}^2 + \hat{u}^2 \propto 1 + (1 - y)^2$$

thus no dependence on azimuthal angle ϕ_h at first order of PT.

The experimental data reveal that
 $d\sigma^{Ip \rightarrow Ih^\pm X} / d\phi_h \propto A + B \cdot \cos(\phi_h) + D \cdot \cos(2\phi_h)$

M. Arneodo et al (EMC): Z. Phys. C 34 (1987) 277



Cahn: the observed azimuthal dependence is related to the intrinsic k_{\perp} of quarks (at least for small P_T values)

$$k_{\perp} = (0, k_{\perp} \cos(\varphi), k_{\perp} \sin(\varphi), 0)$$

$$\hat{s} = sx \left[1 - \frac{2k_{\perp}}{Q} \sqrt{1-y} \cdot \cos(\varphi) \right] + \mathcal{O}\left(\frac{k_{\perp}^2}{Q}\right)$$

$$\hat{u} = sx(1-y) \left[1 - \frac{2k_{\perp}}{Q\sqrt{1-y}} \cdot \cos(\varphi) \right] + \mathcal{O}\left(\frac{k_{\perp}^2}{Q}\right)$$

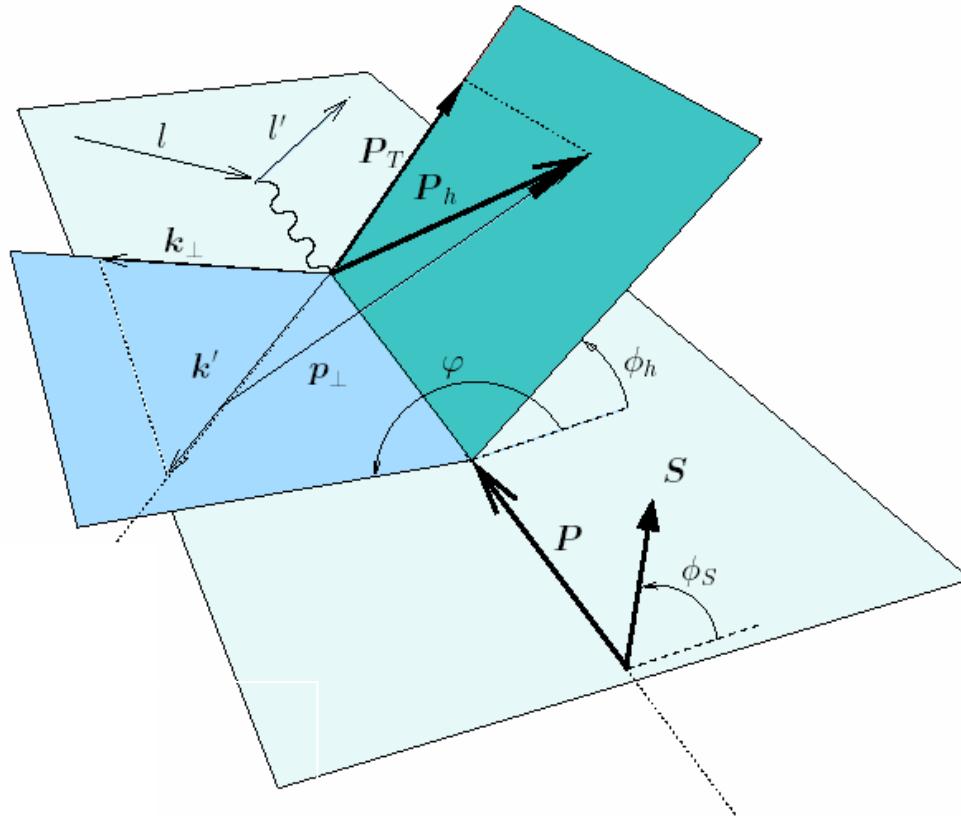


assuming collinear fragmentation, $\varphi = \Phi_h$

$$\frac{d\sigma_{ep \rightarrow ehX}}{d\phi_h} \propto \hat{s}^2 + \hat{u}^2 \propto A + B \cdot \cos(\phi_h) + D \cdot \cos(2\phi_h)$$

These modulations of the cross section with azimuthal angle are denoted as “Cahn effect”.

SIDIS with intrinsic \mathbf{k}_\perp

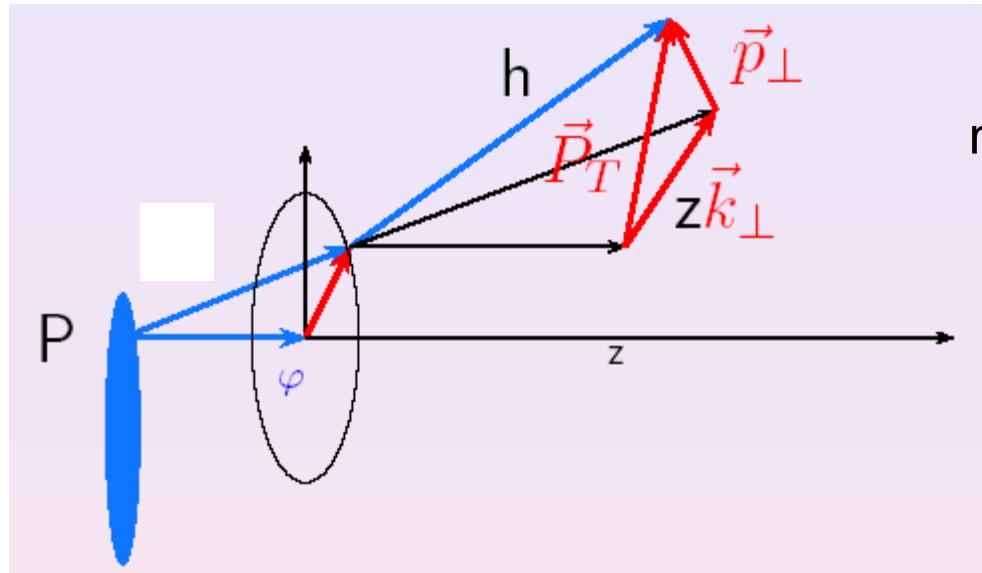


kinematics
according to Trento
conventions (2004)

factorization holds at large Q^2 , and $P_T \approx k_\perp \approx \Lambda_{QCD}$ Ji, Ma, Yuan

$$d\sigma^{lp \rightarrow lhX} = \sum_q f_q(x, k_\perp; Q^2) \otimes d\hat{\sigma}^{lq \rightarrow lq}(y, \vec{k}_\perp; Q^2) \otimes D_q^h(z, p_\perp; Q^2)$$

The situation is more complicated as the produced hadron has also intrinsic transverse momentum with respect to the fragmenting parton.



neglecting terms

$$\mathcal{O}\left(\frac{k_\perp^2}{Q^2}\right)$$

$$\vec{P}_T = \vec{p}_\perp + z \vec{k}_\perp$$

$$\begin{aligned} \frac{d\sigma_{ep \rightarrow ehX}}{d\phi_h} \propto & \int d^2 k_\perp \{ [1 + (1 - y)^2] f_q(x, k_\perp^2) D_h^q(z, (\vec{P}_T - z \vec{k}_\perp)^2) - \\ & - 4\sqrt{1 - y}(2 - y) \frac{k_\perp \cos(\varphi)}{Q} f_q(x, k_\perp^2) D_h^q(z, (\vec{P}_T - z \vec{k}_\perp)^2) \} + \mathcal{O}\left(\frac{k_\perp^2}{Q^2}\right) \end{aligned}$$

assuming:

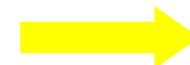
$$\left\{ \begin{array}{l} f_q(x, k_\perp^2) = f_q(x) \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-\frac{k_\perp^2}{\langle k_\perp^2 \rangle}} \\ D_h^q(z, p_\perp^2) = D_h^q(z) \frac{1}{\pi \langle p_\perp^2 \rangle} e^{-\frac{p_\perp^2}{\langle p_\perp^2 \rangle}} \end{array} \right.$$

one finds

$$\frac{d^5 \sigma^{ep \rightarrow ehX}}{dx dy dz P_T dP_T d\phi_h} \propto \{ [1 + (1 - y)^2] - 4 \frac{\sqrt{1 - y}(2 - y)\langle k_\perp^2 \rangle z P_T}{(\langle p_\perp^2 \rangle + z^2 \langle k_\perp^2 \rangle) Q} \cos(\phi_h) \} \\ \cdot f_q(x) D_h^q(z) \frac{1}{\pi \langle P_T^2 \rangle} e^{-\frac{P_T^2}{\langle P_T^2 \rangle}}$$

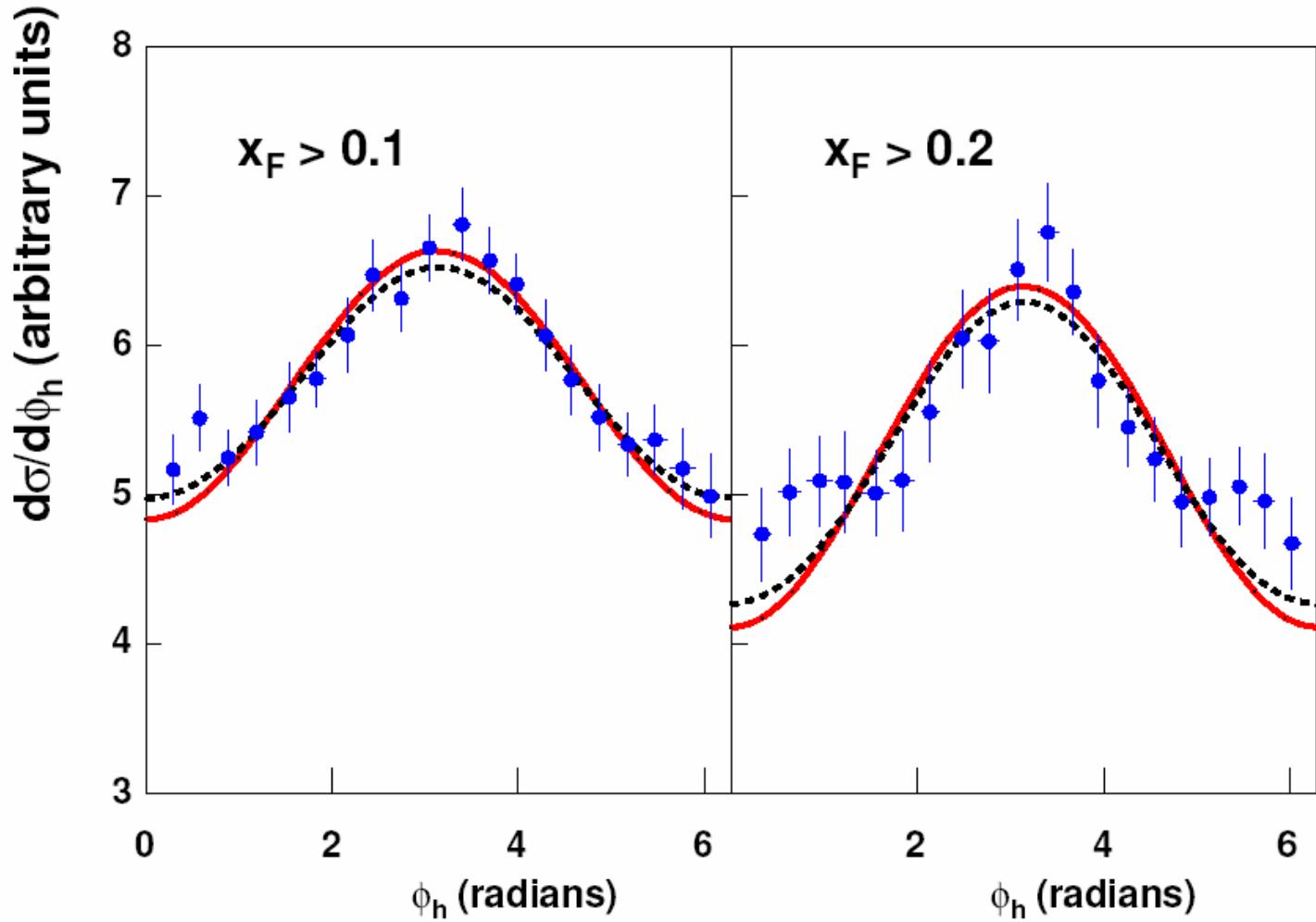
with

$$\langle P_T^2 \rangle = \langle p_\perp^2 \rangle + z^2 \langle k_\perp^2 \rangle$$

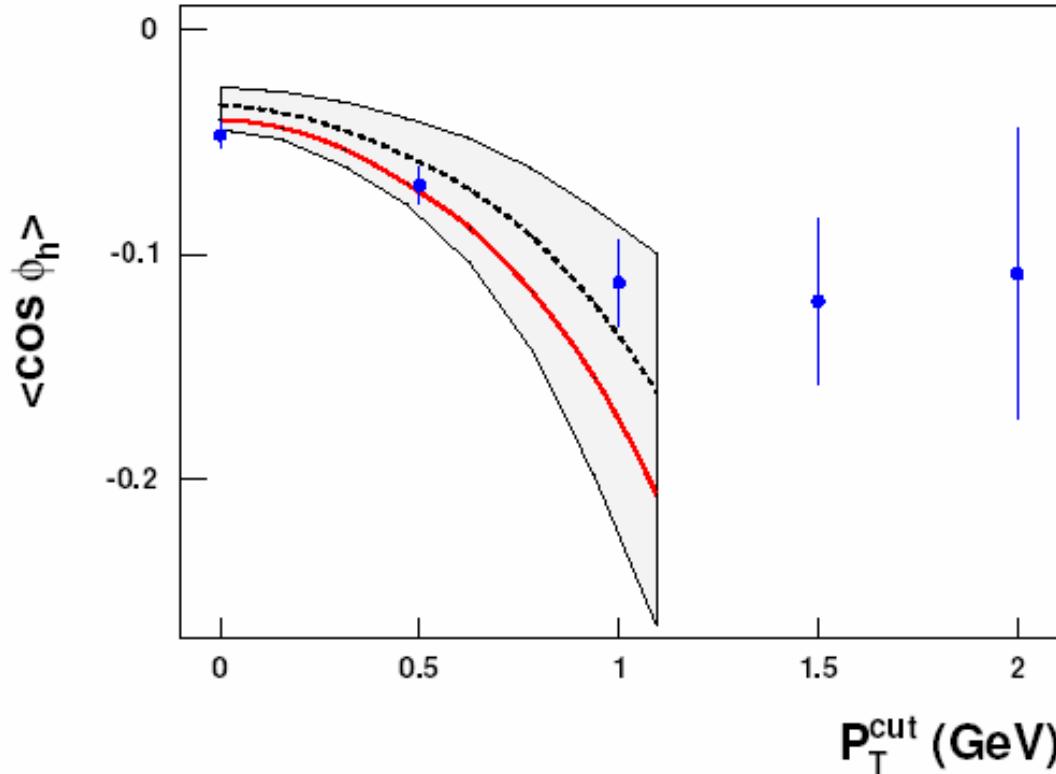


clear dependence on $\langle p_\perp^2 \rangle$ & $\langle k_\perp^2 \rangle$ (assumed to be constant)

Find best values by fitting data on Φ_h and P_T dependences



EMC data, μp and μd , E between 100 and 280 GeV. Dashed line = exact kinematics, red solid line = only terms up to $O(k_\perp/Q)$



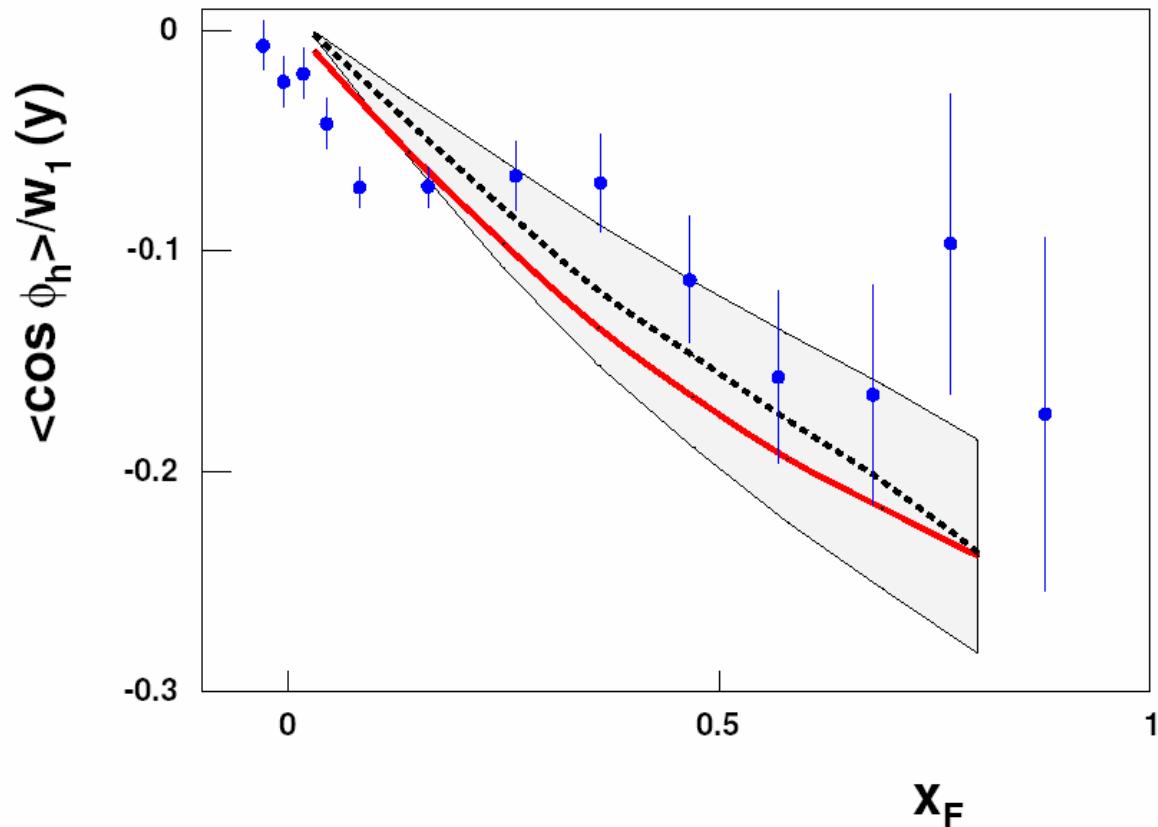
$$\langle \cos(\phi_h) \rangle = \frac{\int \sigma \cos(\phi_h) d\phi_h}{\int \sigma d\phi_h}$$

The closed area shows effects of varying by 20%

$$\langle p_\perp^2 \rangle \text{ & } \langle k_\perp^2 \rangle$$

Data from E665, $E_{\text{Lab}} = 490 \text{ GeV}$. σ is integrated from P_T^{cut} to P_T^{max} . At low P_T^{cut} the non perturbative k_\perp contributions dominate. At large P_T^{cut} NLO pQCD contributions take over ($\gamma * g \rightarrow q \bar{q}$, $\gamma * q \rightarrow q g$)

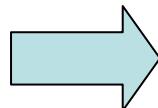
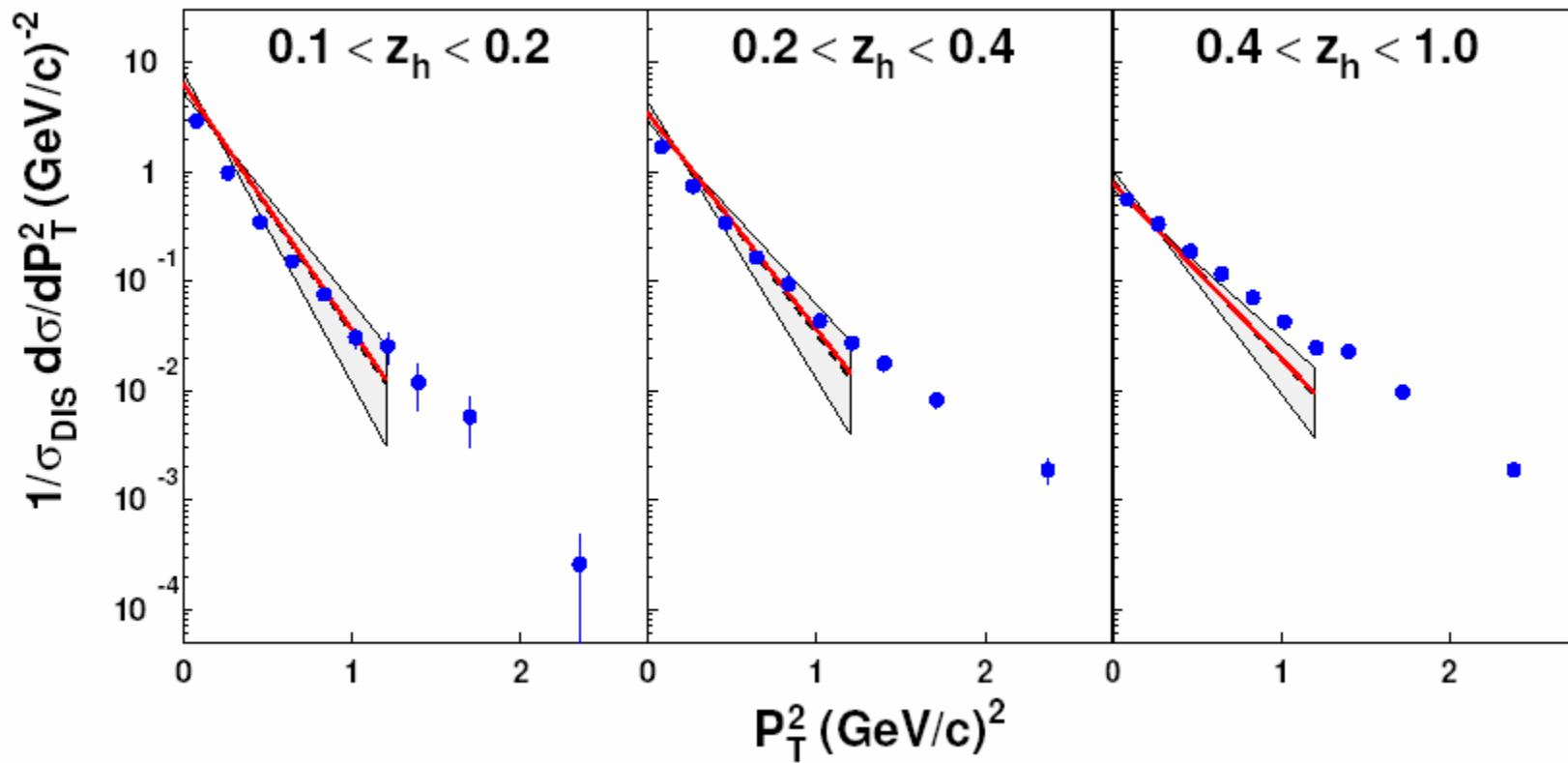
EMC data



$$\langle \cos(\phi_h) \rangle = \frac{\int \sigma \cos(\phi_h) d\phi_h}{\int \sigma d\phi_h}$$

$$w_1(y) = \frac{(2-y)\sqrt{1-y}}{1+(1-y)^2}$$

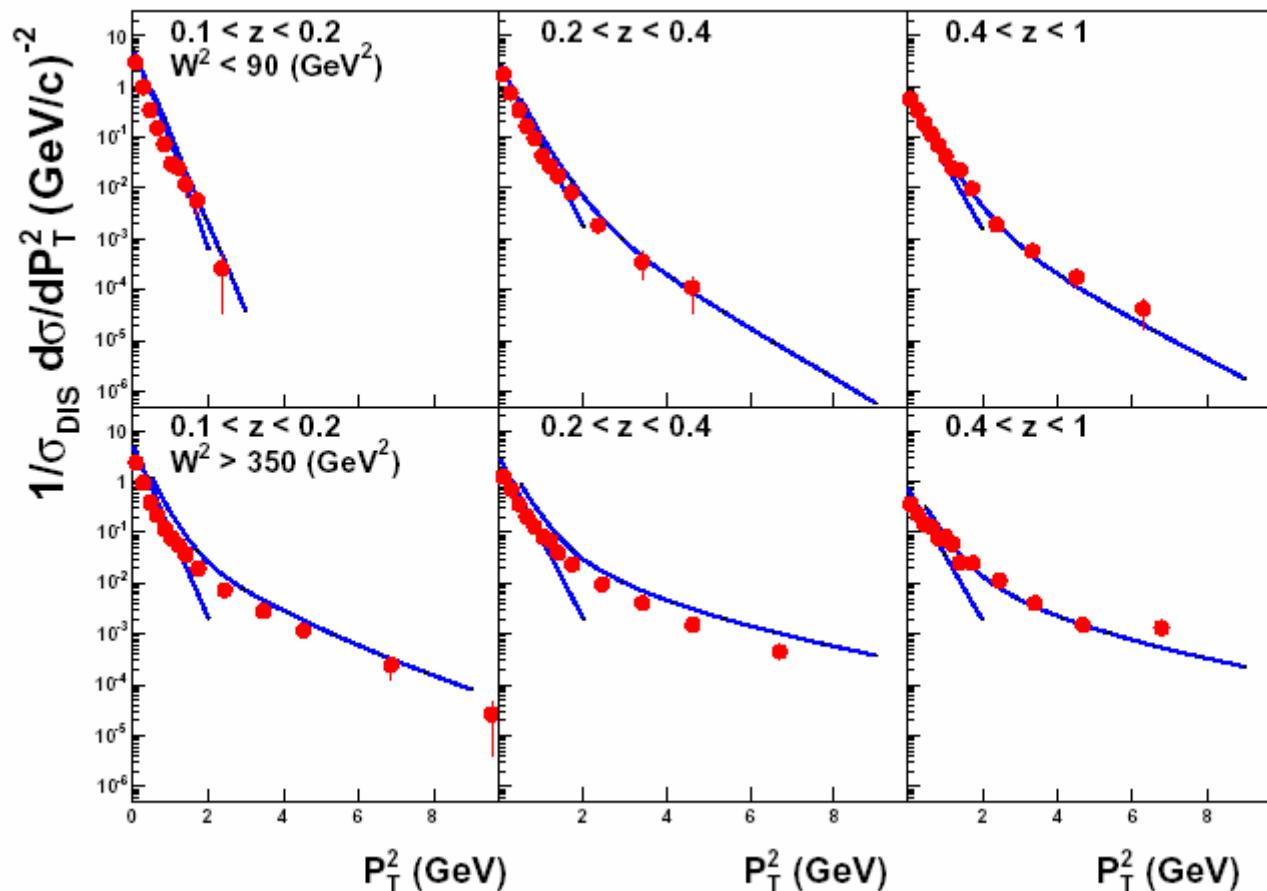
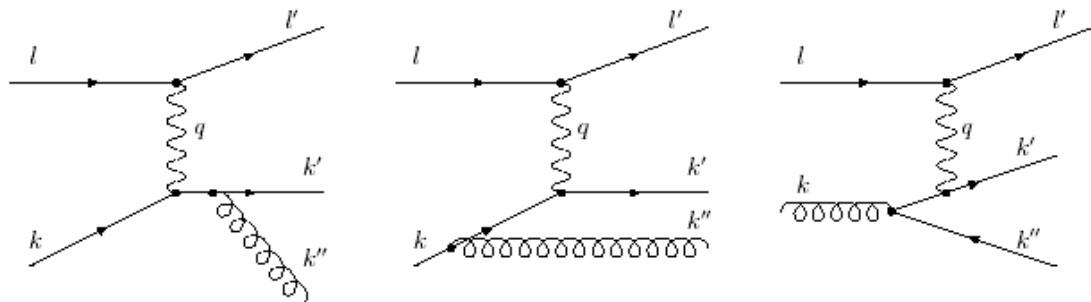
EMC data

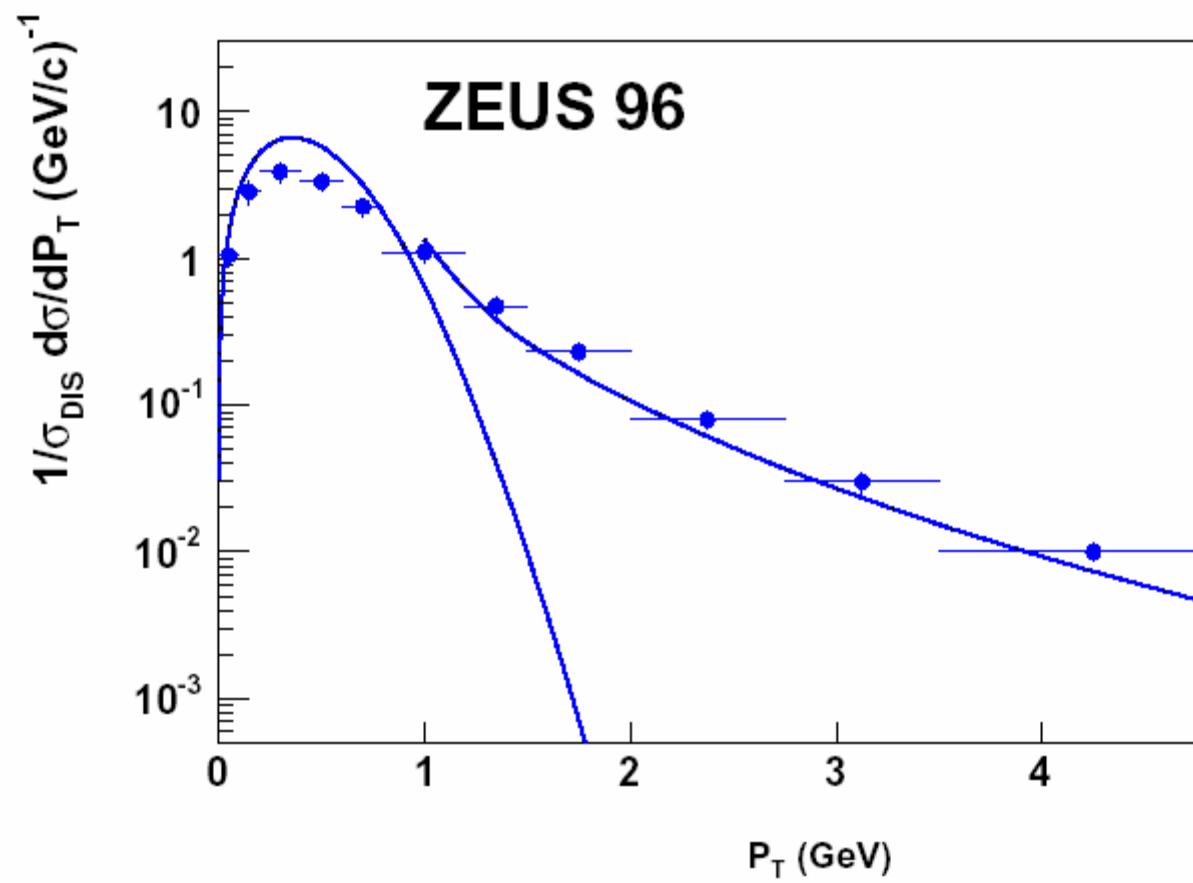


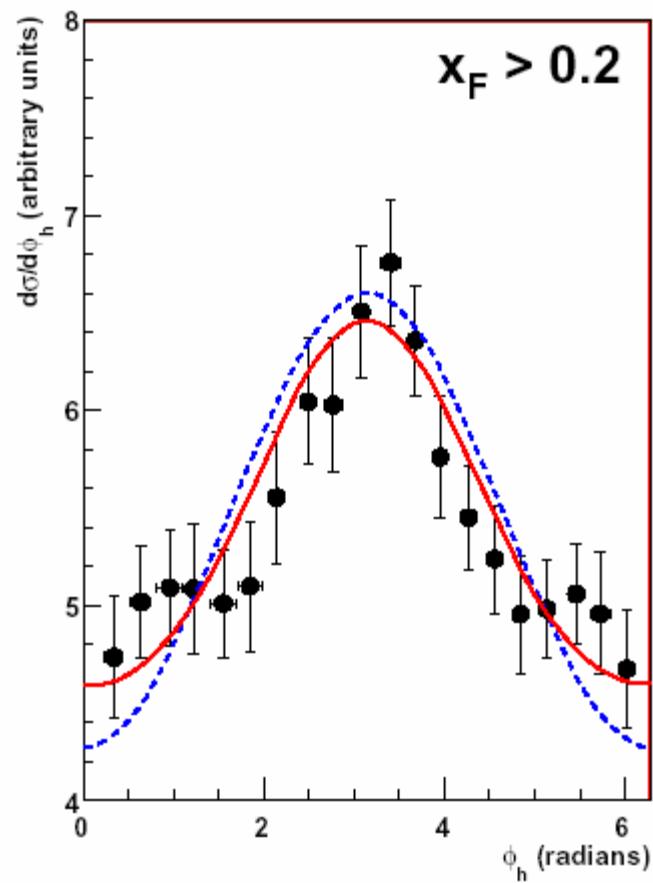
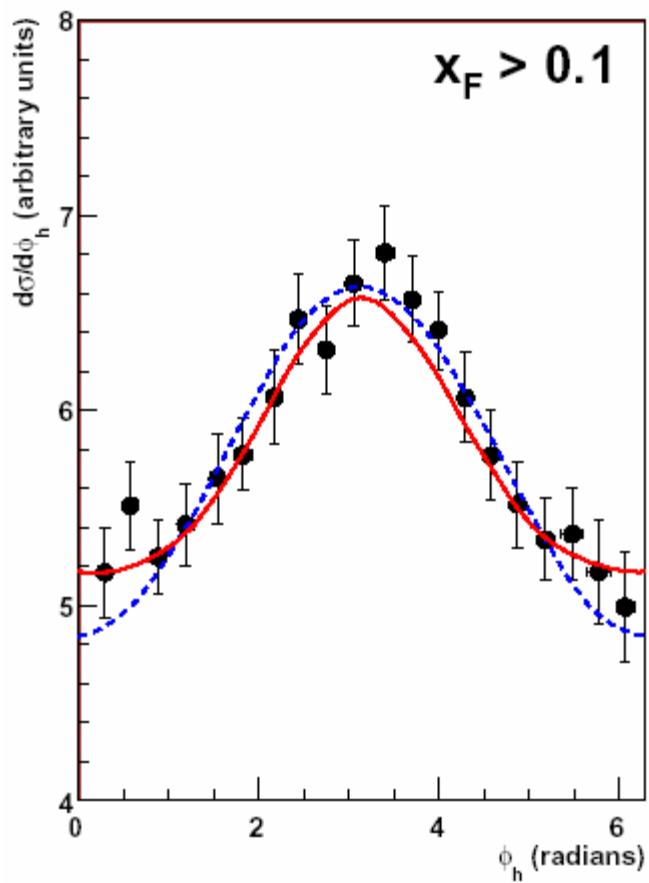
Fitting the unpolarized data
leads to the best values

$$\langle k_\perp^2 \rangle = 0.25 \text{ GeV}^2$$
$$\langle p_\perp^2 \rangle = 0.2 \text{ GeV}^2$$

Large P_T data explained
by NLO QCD
corrections

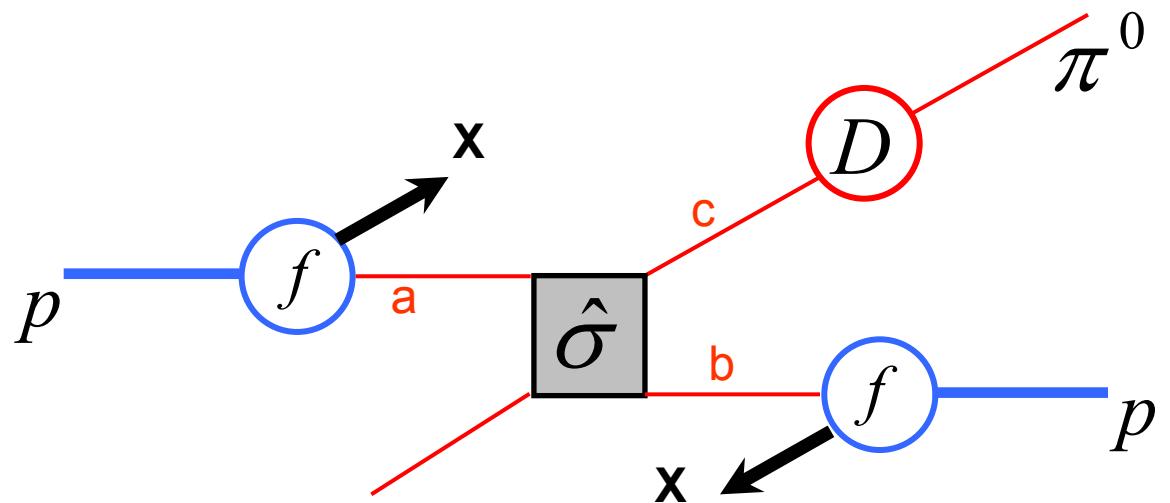






$pp \rightarrow \pi^0 X$ (collinear configurations)

factorization theorem



$$d\sigma = \sum_{a,b,c,d=q,\bar{q},g} \underbrace{f_{a/p} \otimes f_{b/p}}_{\text{PDF}} \otimes d\hat{\sigma}^{ab \rightarrow cd} \otimes \underbrace{D_{\pi/c}}_{\text{FF}}$$

pQCD elementary interactions

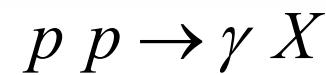
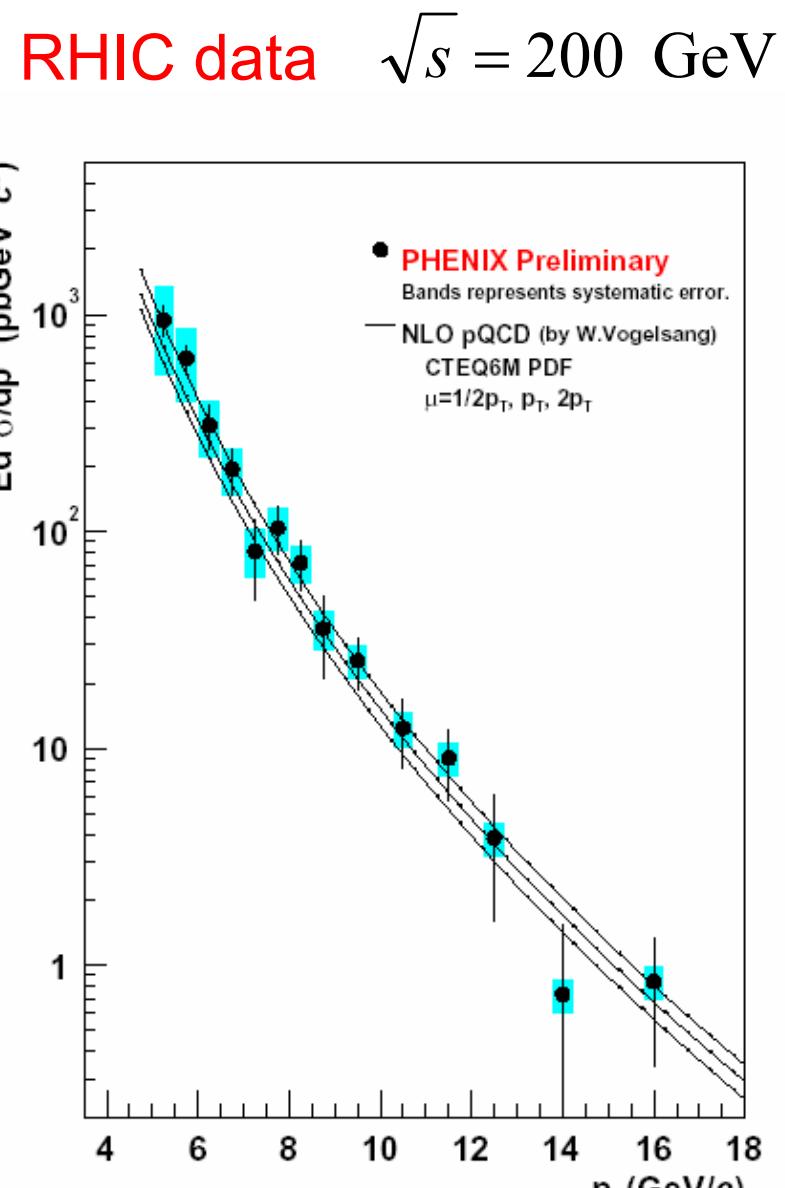
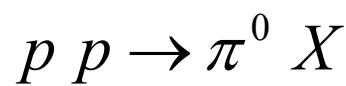
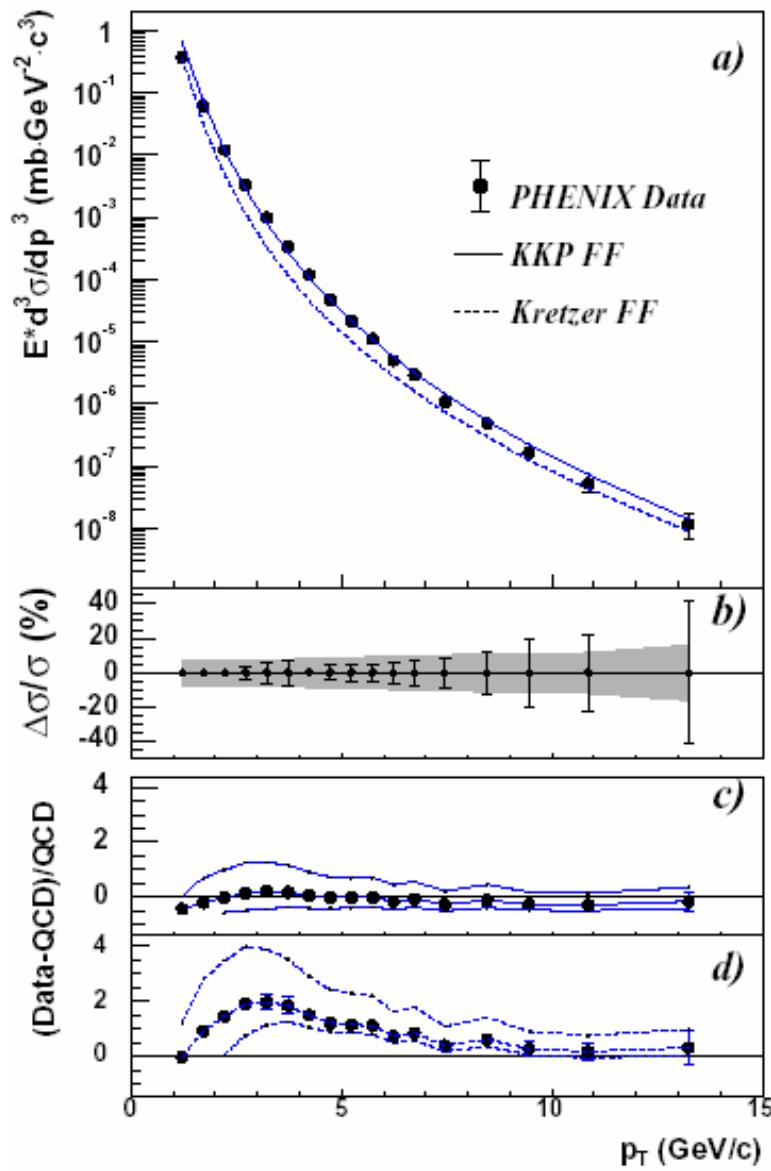
The cross section

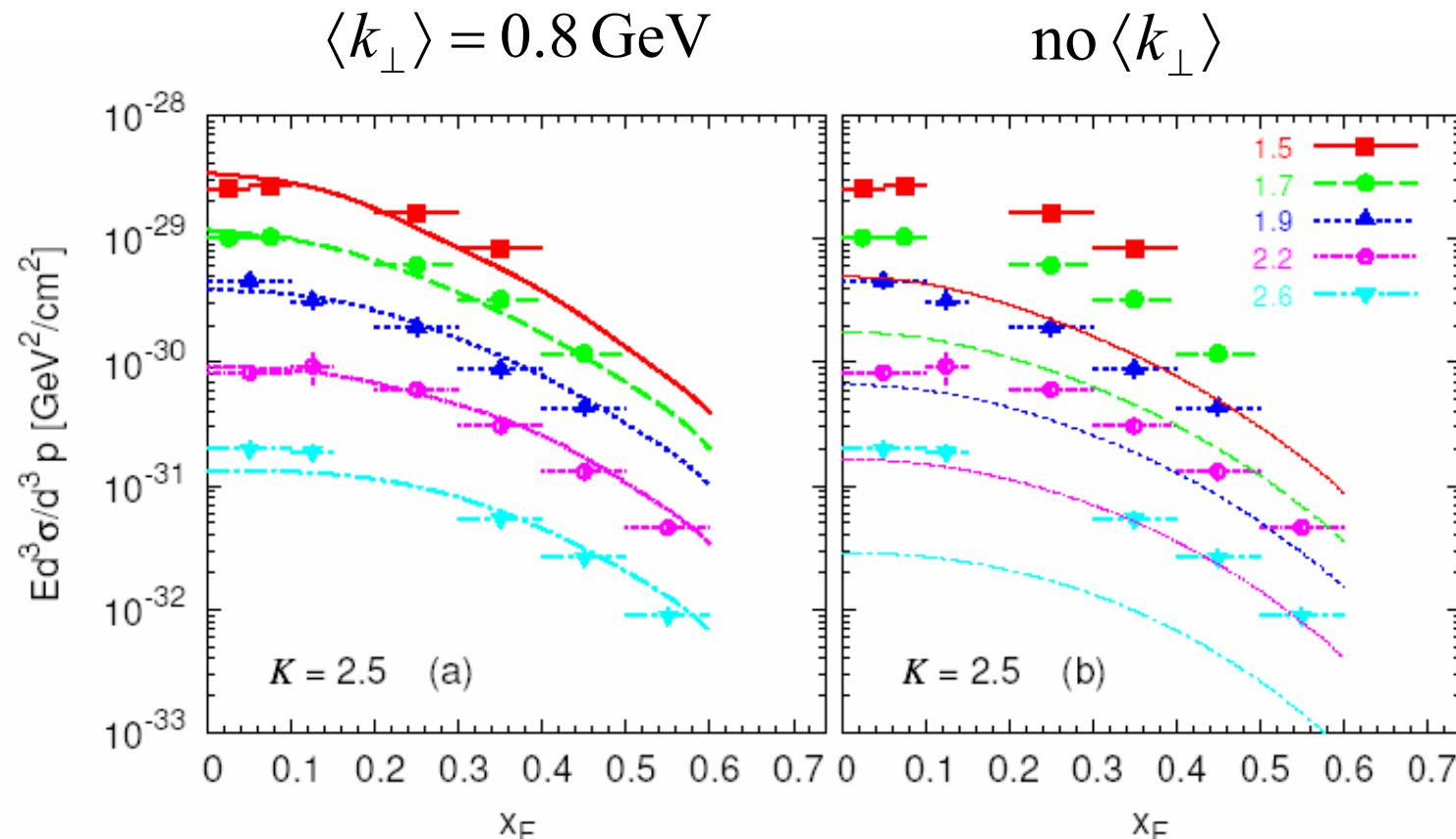
$$\begin{aligned}
\frac{E_C d\sigma^{AB \rightarrow CX}}{d^3 p_C} &= \sum_{a,b,c,d} \int dx_a dx_b dz f_{a/A}(x_a, Q^2) f_{b/B}(x_b, Q^2) \\
&\times \frac{\hat{s}}{\pi z^2} \frac{d\hat{\sigma}^{ab \rightarrow cd}}{d\hat{t}}(\hat{s}, \hat{t}, \hat{u}, x_a, x_b) \delta(\hat{s} + \hat{t} + \hat{u}) D_{C/c}(z, Q^2) \\
&= \sum_{a,b,c,d} \int dx_a dx_b f_{a/A}(x_a, Q^2) f_{b/B}(x_b, Q^2) \\
&\times \frac{1}{\pi z} \frac{d\hat{\sigma}^{ab \rightarrow cd}}{d\hat{t}}(\hat{s}, \hat{t}, \hat{u}, x_a, x_b) D_{C/c}(z, Q^2),
\end{aligned}$$

$$x_a x_b ZS = -x_a t - x_b u$$

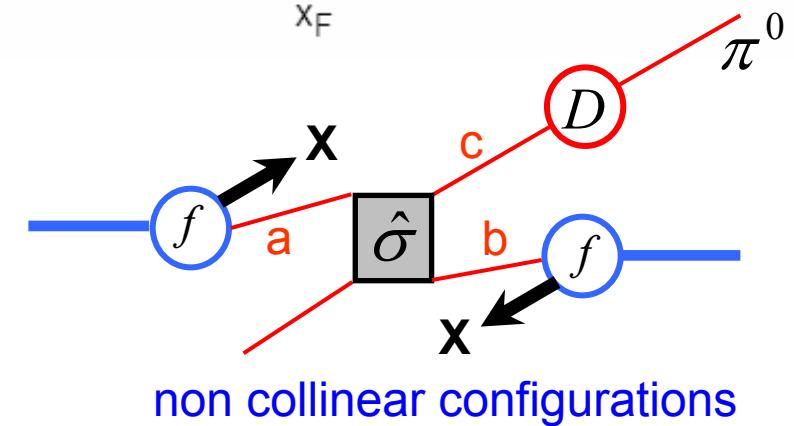
$\hat{s}, \hat{t}, \hat{u}$ elementary Mandelstam variables

s, t, u hadronic Mandelstam variables

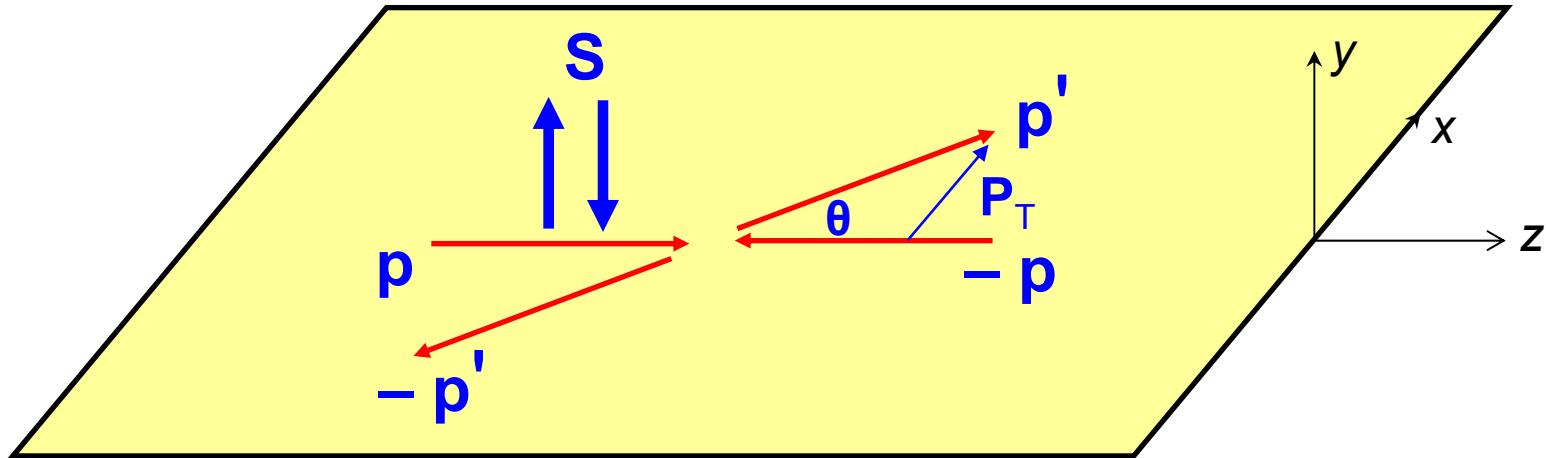




F. Murgia, U. D'Alesio
 FNAL data, PLB 73 (1978)
 $p \ p \rightarrow \pi^0 \ X \quad \sqrt{s} \approx 20 \text{ GeV}$
 original idea by Feynman-Field



Transverse single spin asymmetries: elastic scattering



$$A_N \equiv \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \propto \vec{S} \cdot (\vec{p} \times \vec{P}_T) \propto \sin \theta$$

$$M_{++;++} \equiv \Phi_1$$

$$M_{--;++} \equiv \Phi_2$$

Example: $pp \rightarrow pp$ →

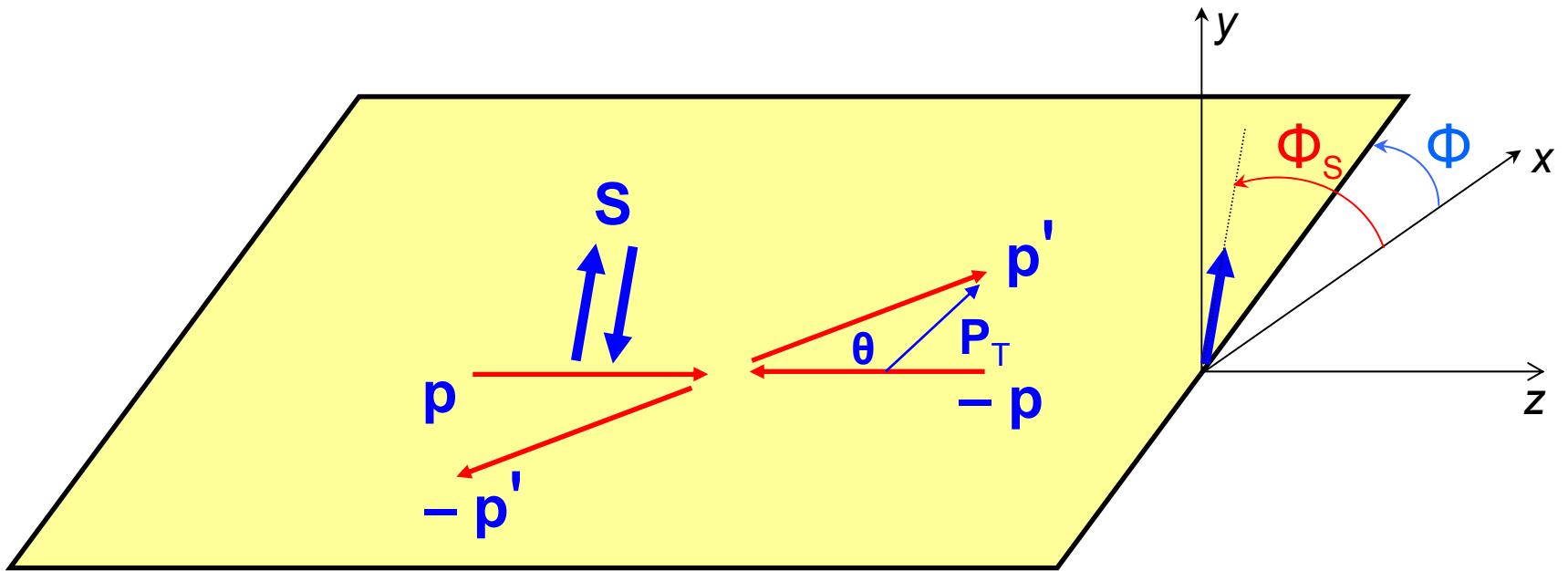
$$M_{+-;+-} \equiv \Phi_3$$

$$M_{-+;+-} \equiv \Phi_4$$

5 independent helicity amplitudes

$$A_N \propto \text{Im} [\Phi_5 (\Phi_1 + \Phi_2 + \Phi_3 - \Phi_4)^*]$$

$$M_{-+;++} \equiv \Phi_5$$

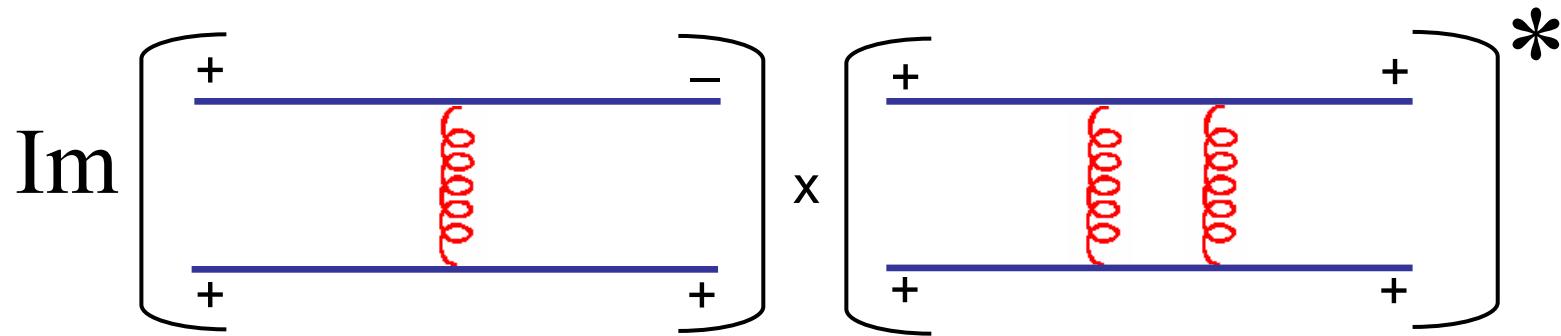


for a generic configuration:

$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \propto \vec{S} \cdot (\vec{p} \times \vec{P}_T) \propto P_T \sin(\Phi_S - \Phi)$$

Single spin asymmetries at partonic level. Example: $qq' \rightarrow qq'$

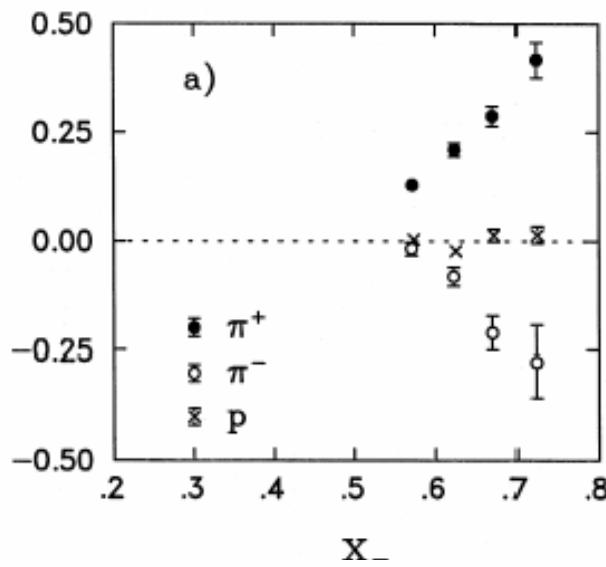
$$A_N \neq 0 \quad \text{needs helicity flip + relative phase}$$



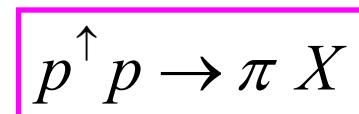
QED and QCD interactions conserve helicity, up to corrections $O(m_q / E)$

→ $A_N \propto \frac{m_q}{E} \alpha_s$ at quark level

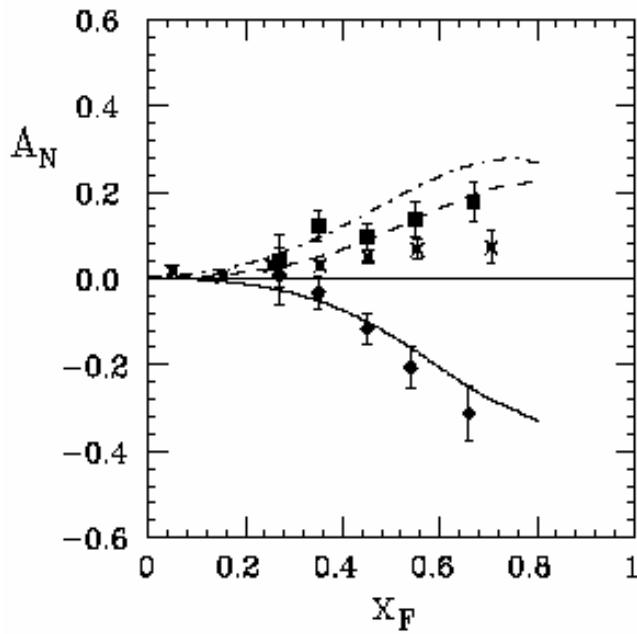
but large SSA observed at hadron level!



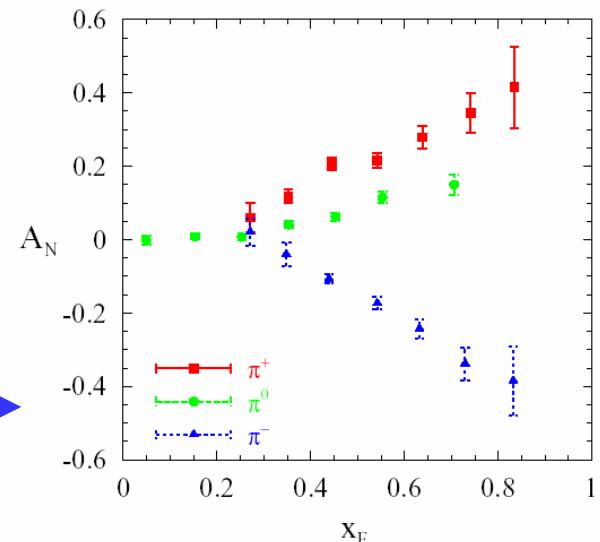
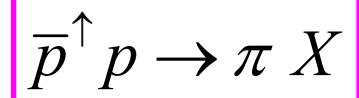
BNL-AGS $\sqrt{s} = 6.6 \text{ GeV}$
 $0.6 < p_T < 1.2$



E704 $\sqrt{s} = 20 \text{ GeV}$
 $0.7 < p_T < 2.0$



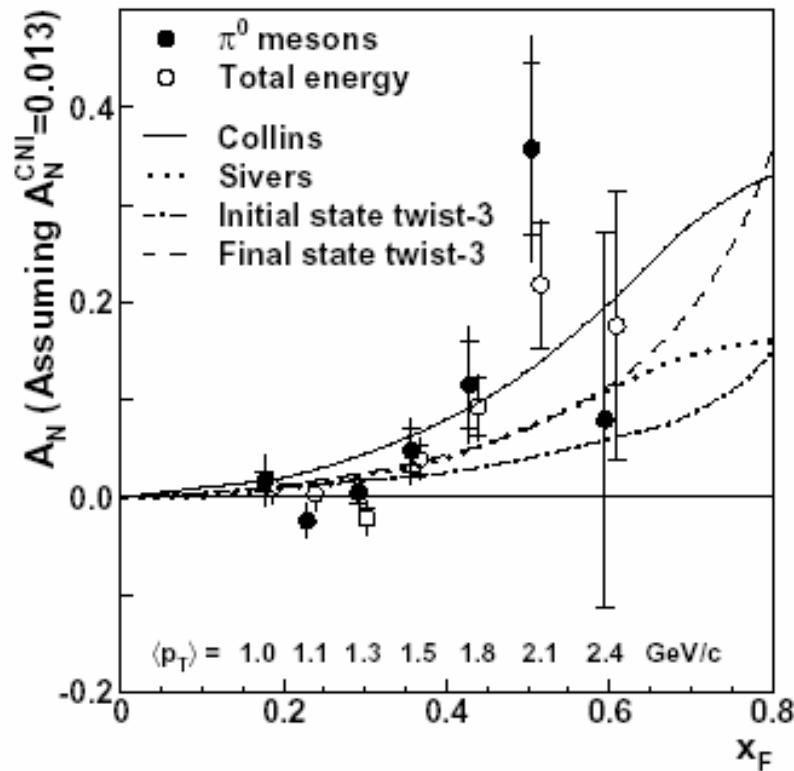
E704 $\sqrt{s} = 20 \text{ GeV}$
 $0.7 < p_T < 2.0$



observed transverse Single Spin Asymmetries

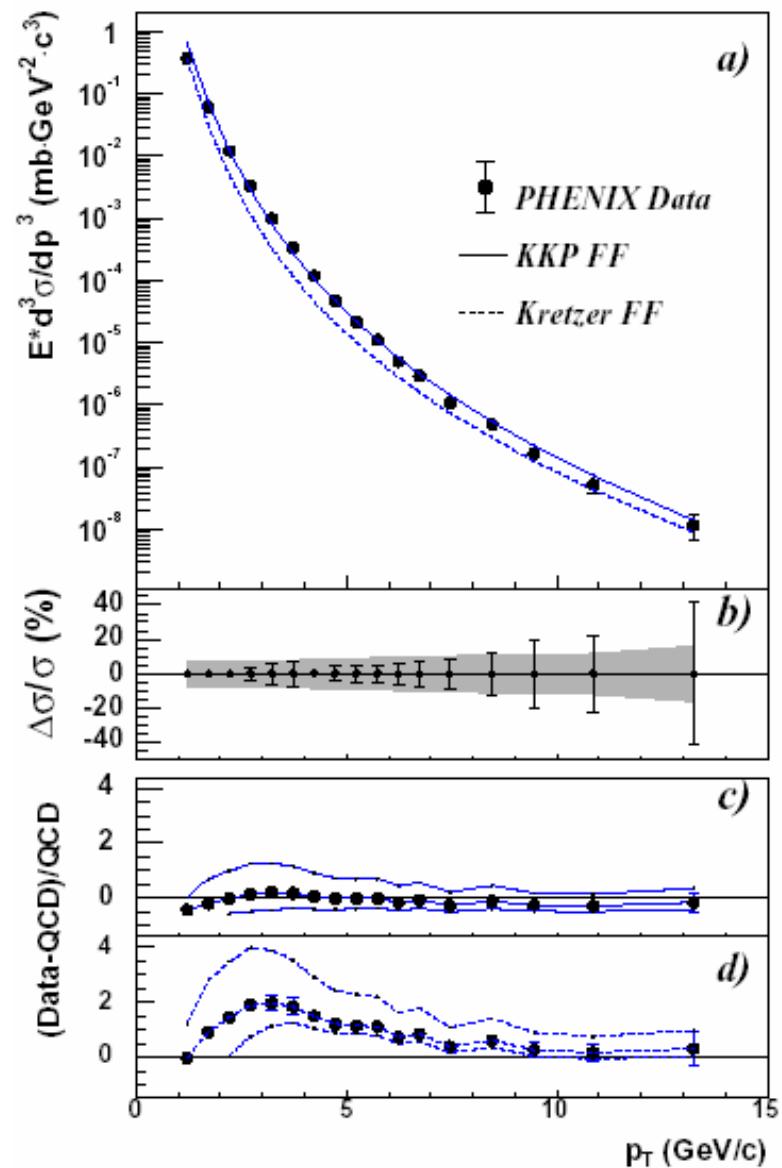
$$A_N \equiv \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

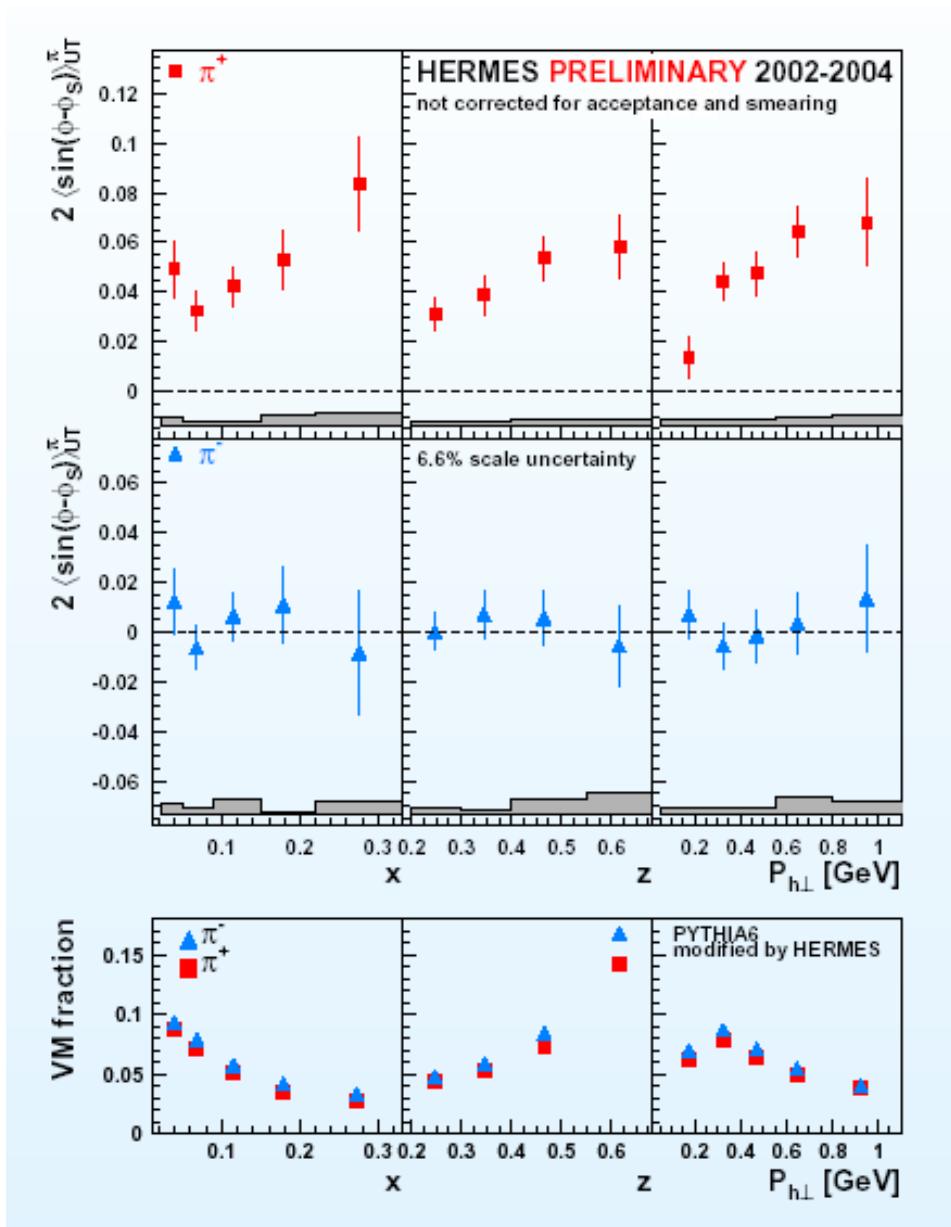
experimental data on SSA



STAR-RHIC $\sqrt{s} = 200$ GeV
 $1.1 < p_T < 2.5$

A_N stays at high energies





$$l N^\uparrow \rightarrow l \pi X$$

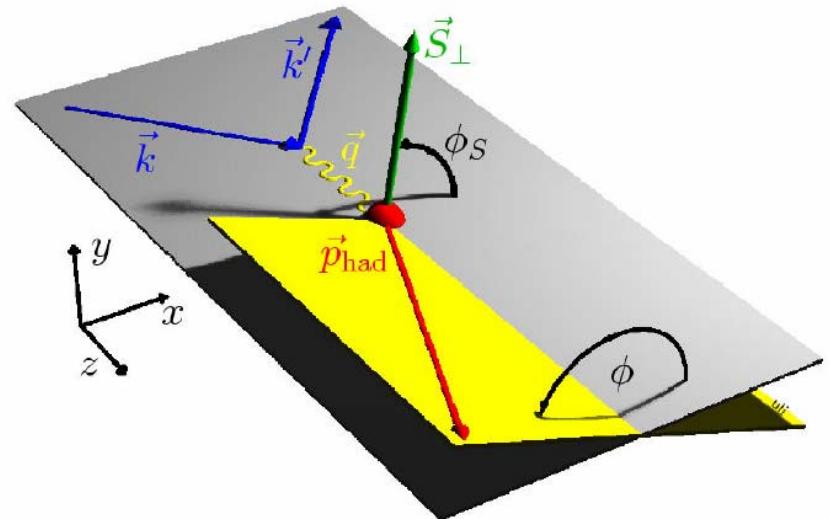


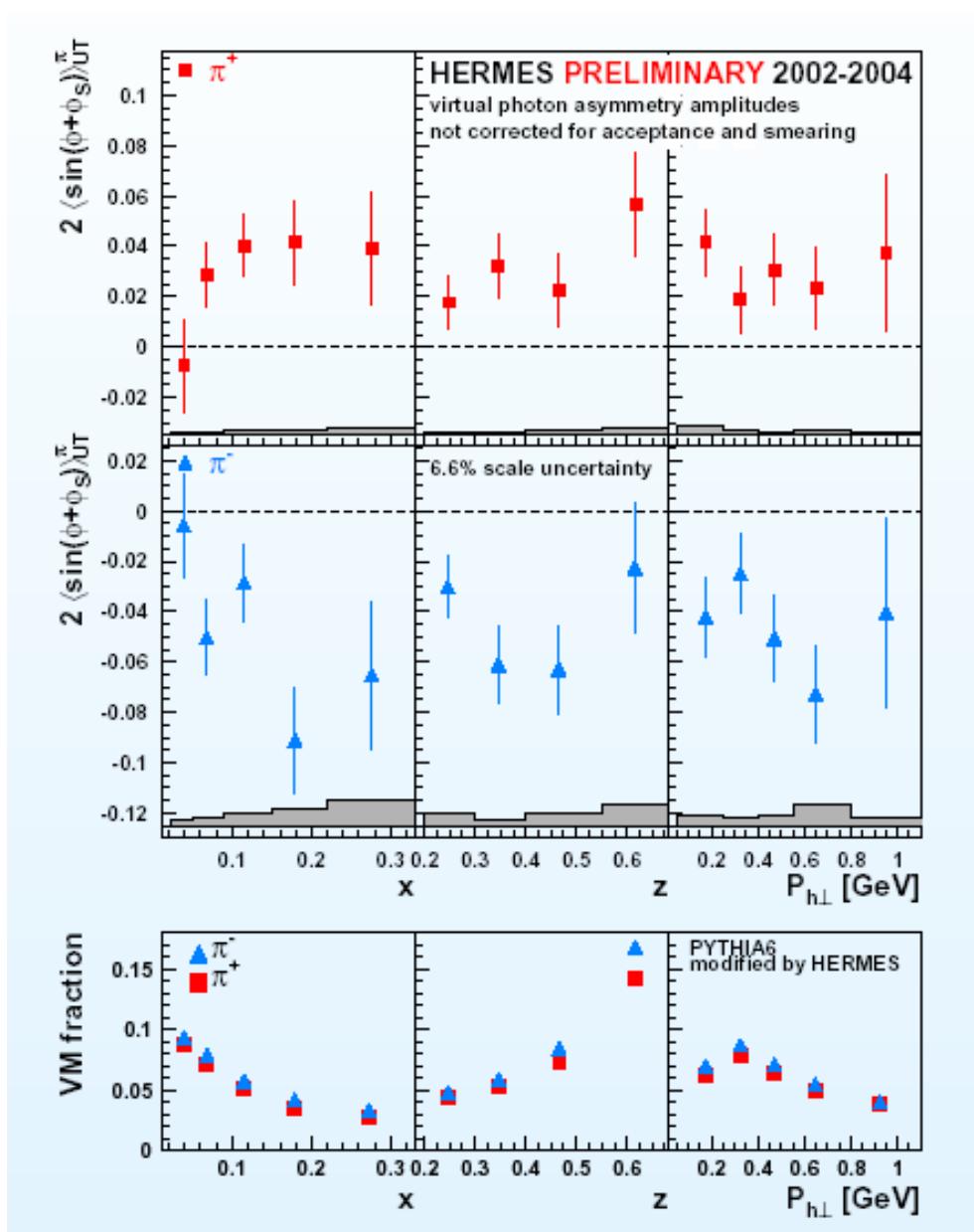
“Sivers moment”

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

$$2\langle \sin(\Phi - \Phi_S) \rangle = A_{UT}^{\sin(\Phi - \Phi_S)}$$

$$\equiv 2 \frac{\int d\Phi d\Phi_S (d\sigma^\uparrow - d\sigma^\downarrow) \sin(\Phi - \Phi_S)}{\int d\Phi d\Phi_S (d\sigma^\uparrow + d\sigma^\downarrow)}$$





$$l N^\uparrow \rightarrow l \pi X$$

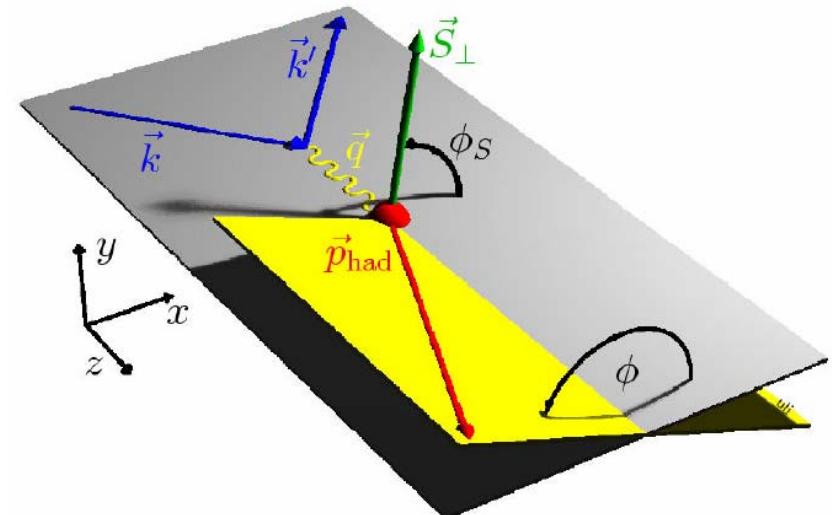


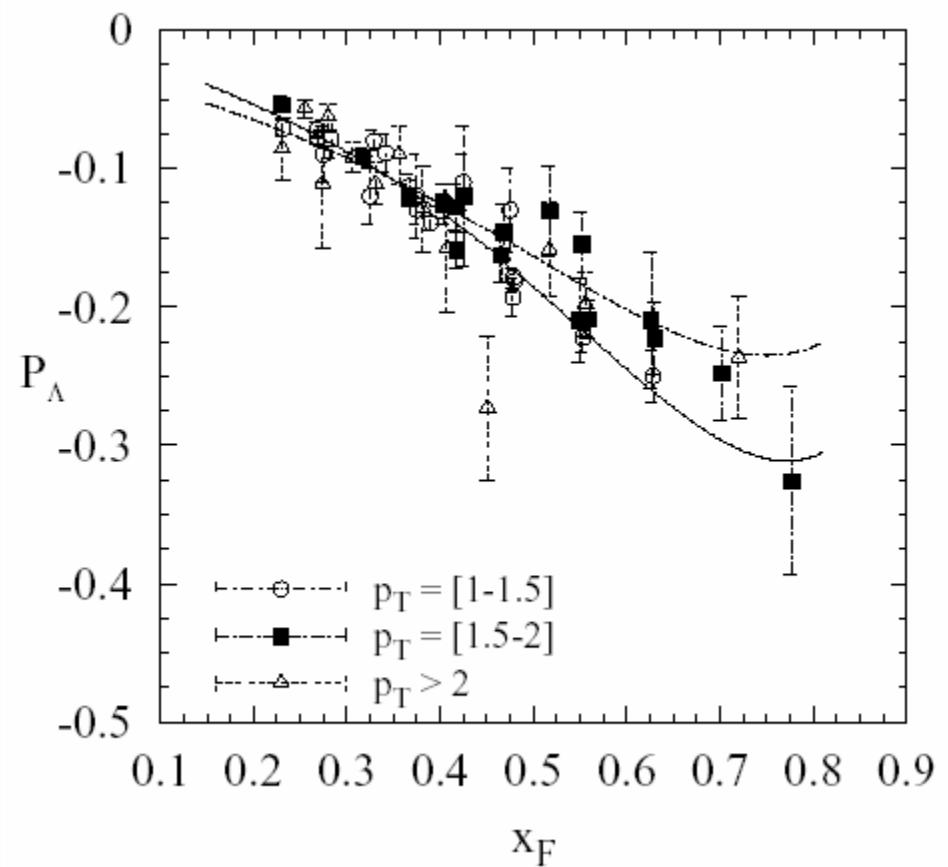
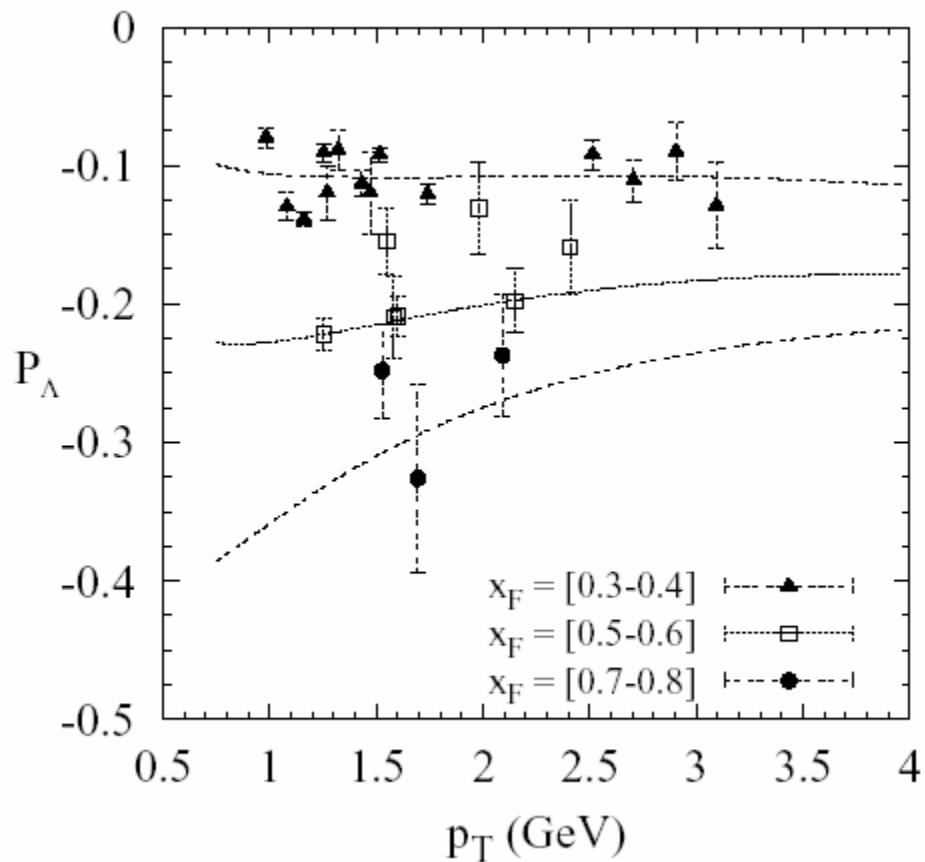
“Collins moment”

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

$$2\langle \sin(\Phi + \Phi_S) \rangle = A_{UT}^{\sin(\Phi + \Phi_S)}$$

$$\equiv 2 \frac{\int d\Phi d\Phi_S (d\sigma^\uparrow - d\sigma^\downarrow) \sin(\Phi + \Phi_S)}{\int d\Phi d\Phi_S (d\sigma^\uparrow + d\sigma^\downarrow)}$$

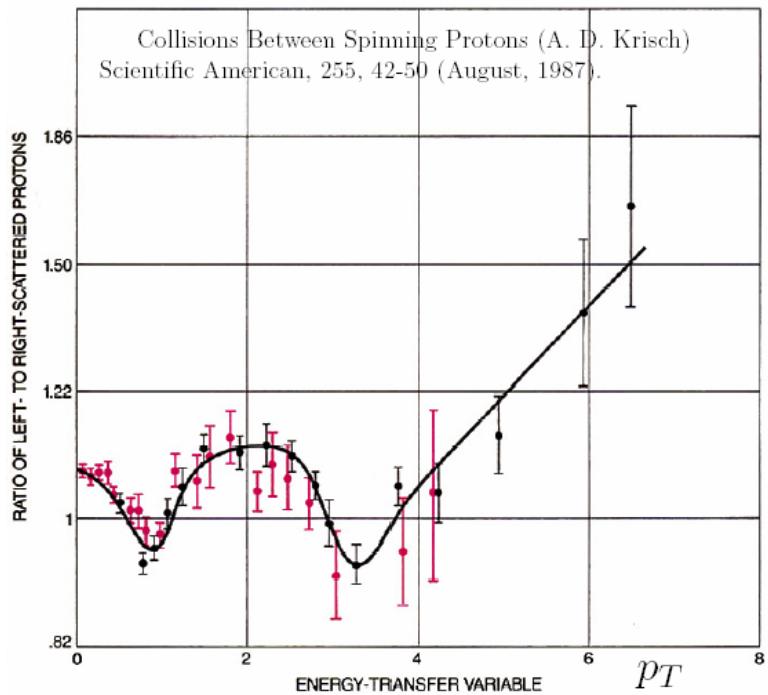
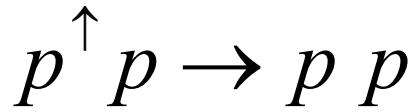




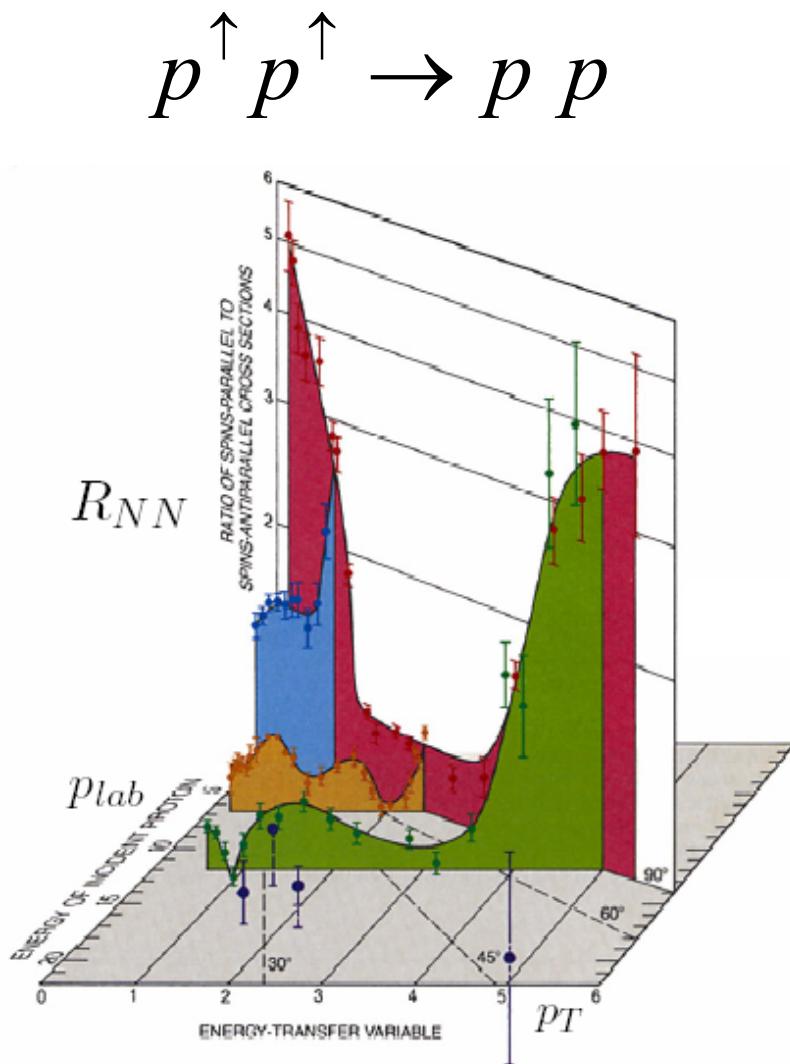
Transverse Λ polarization in unpolarized p-Be scattering at Fermilab

$$p N \rightarrow \Lambda^\uparrow X$$

$$P_\Lambda = \frac{d\sigma^{\Lambda^\uparrow} - d\sigma^{\Lambda^\downarrow}}{d\sigma^{\Lambda^\uparrow} + d\sigma^{\Lambda^\downarrow}}$$

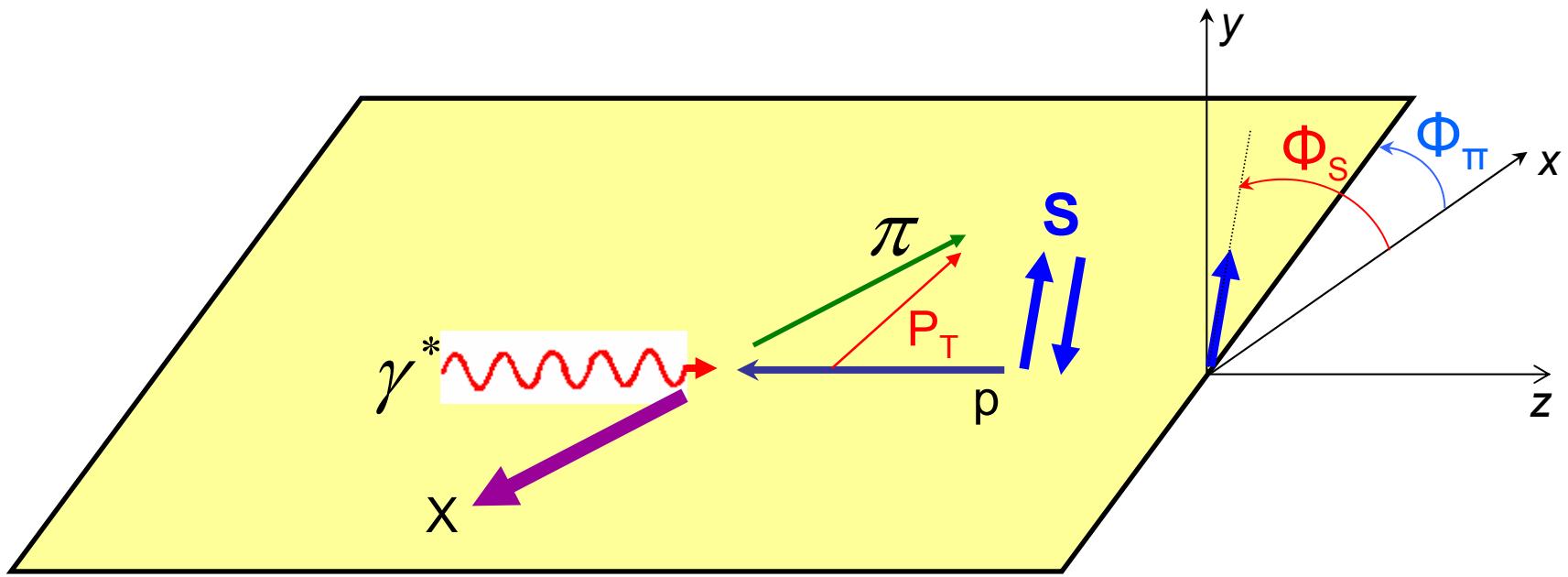


$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$



$$A_{NN} = \frac{d\sigma^{\uparrow\uparrow} - d\sigma^{\uparrow\downarrow}}{d\sigma^{\uparrow\uparrow} + d\sigma^{\uparrow\downarrow}}$$

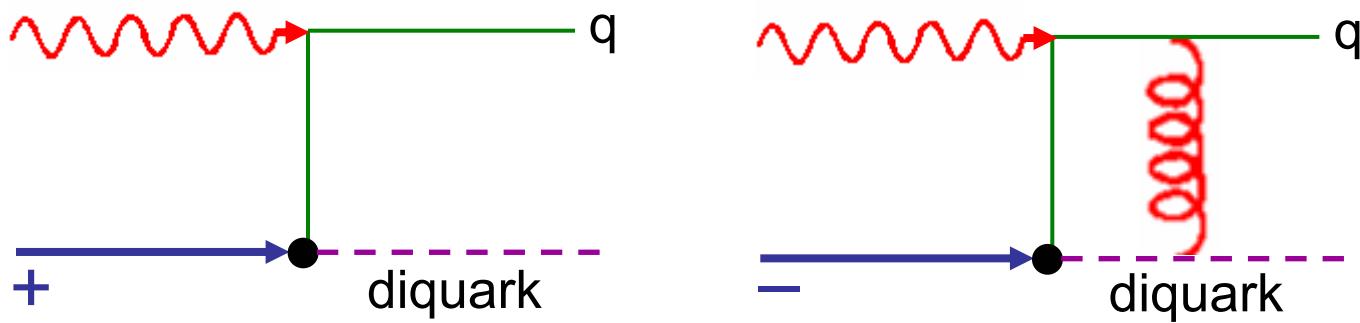
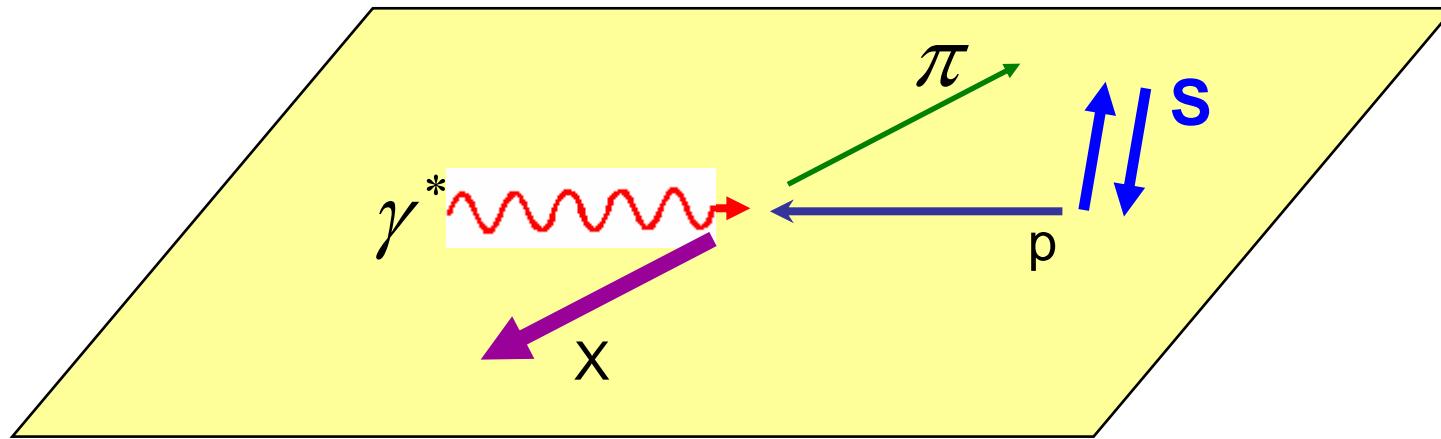
Transverse single spin asymmetries in SIDIS



$$A_N \propto \vec{S} \cdot (\vec{p} \times \vec{P}_T) \propto P_T \sin(\Phi_\pi - \Phi_S) \quad \gamma^* - p \text{ c.m. frame}$$

in collinear configurations there cannot be (at LO) any P_T
 needs \mathbf{k}_\perp dependent quark distribution in p^\uparrow (Sivers mechanism) or
 \mathbf{p}_\perp dependent fragmentation of polarized quark (Collins mechanism)

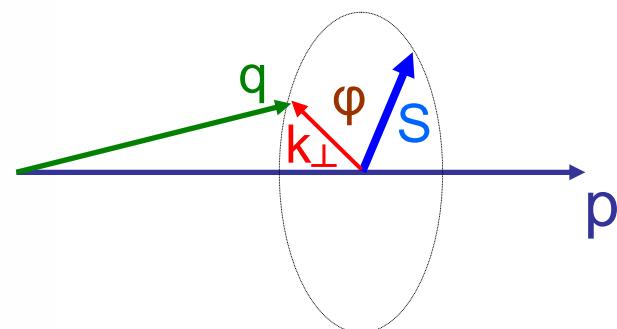
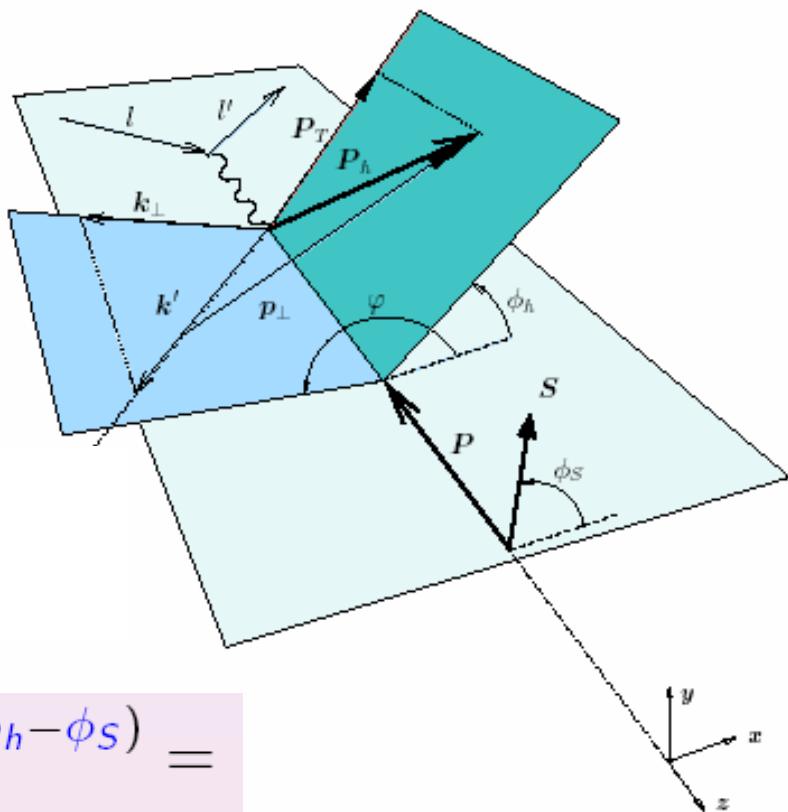
Brodsky, Hwang, Schmidt model for Sivers function



$$\vec{S} \cdot (\vec{p} \times \vec{P}_T) \propto P_T \sin(\Phi_\pi - \Phi_S)$$

needs k_\perp dependent quark distribution in p^\uparrow : Sivers function

Sivers mechanism in SIDIS



$$f_{q/p^\uparrow}(x, \vec{k}_\perp) = f_{q/p}(x, k_\perp) + \frac{1}{2} \Delta^N f_{q/p^\uparrow}(x, k_\perp) \vec{S} \cdot (\hat{p} \times \hat{k}_\perp)$$

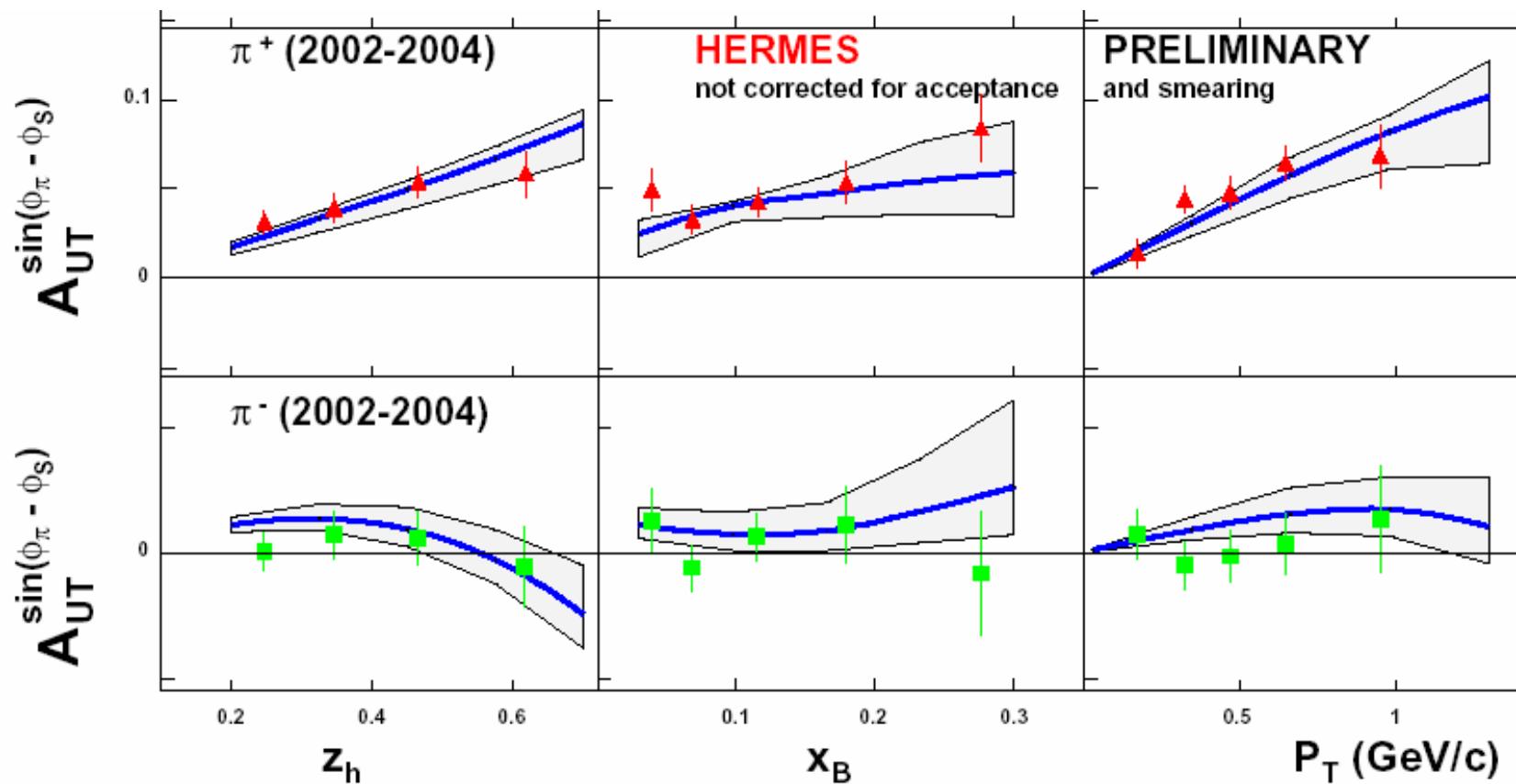
$$A_{UT}^{\sin(\phi_h - \phi_S)} =$$

$$\mathbf{p}_\perp = \mathbf{P}_T - z \mathbf{k}_\perp + \mathcal{O}(k_\perp^2/Q^2)$$

$$\frac{\sum_q d\{\phi_h \phi_S \mathbf{k}_\perp\} \Delta^N f_{q/p^\uparrow}(x, \mathbf{k}_\perp) \sin(\varphi - \phi_S) \frac{d\hat{\sigma}^{\ell q \rightarrow \ell q}}{dQ^2} J \frac{z}{z_h} D_q^h(z, \mathbf{p}_\perp) \sin(\phi_h - \phi_S)}{2 \pi \sum_q d\phi_h d^2 \mathbf{k}_\perp f_q(x, \mathbf{k}_\perp) \frac{d\hat{\sigma}^{\ell q \rightarrow \ell q}}{dQ^2} J \frac{z}{z_h} D_q^h(z, \mathbf{p}_\perp)}$$

$A_{UT}^{\sin(\Phi - \Phi_S)}$ from Sivers mechanism

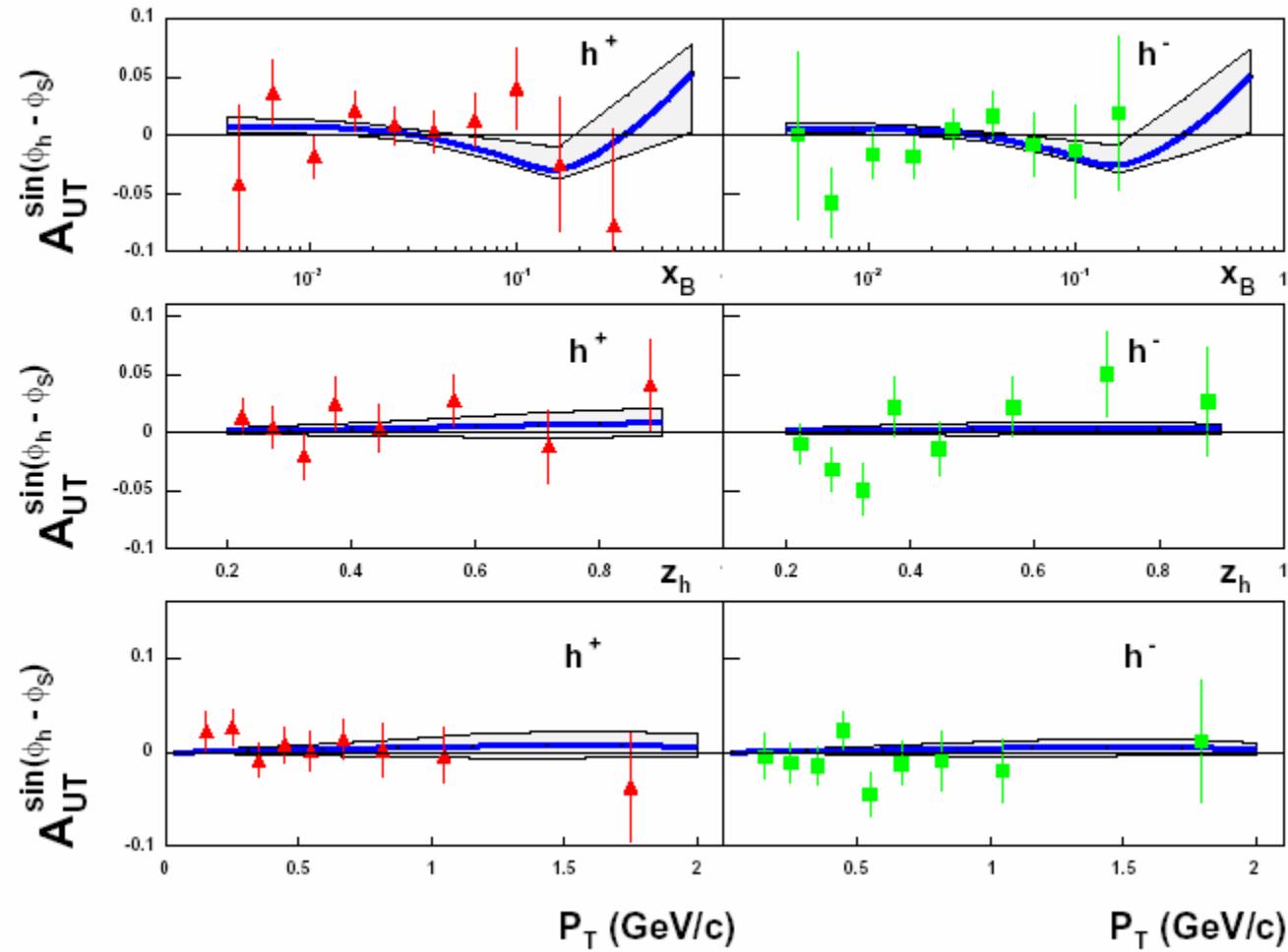
M.A., U.D'Alesio, M.Boglione, A.Kotzinian, A Prokudin



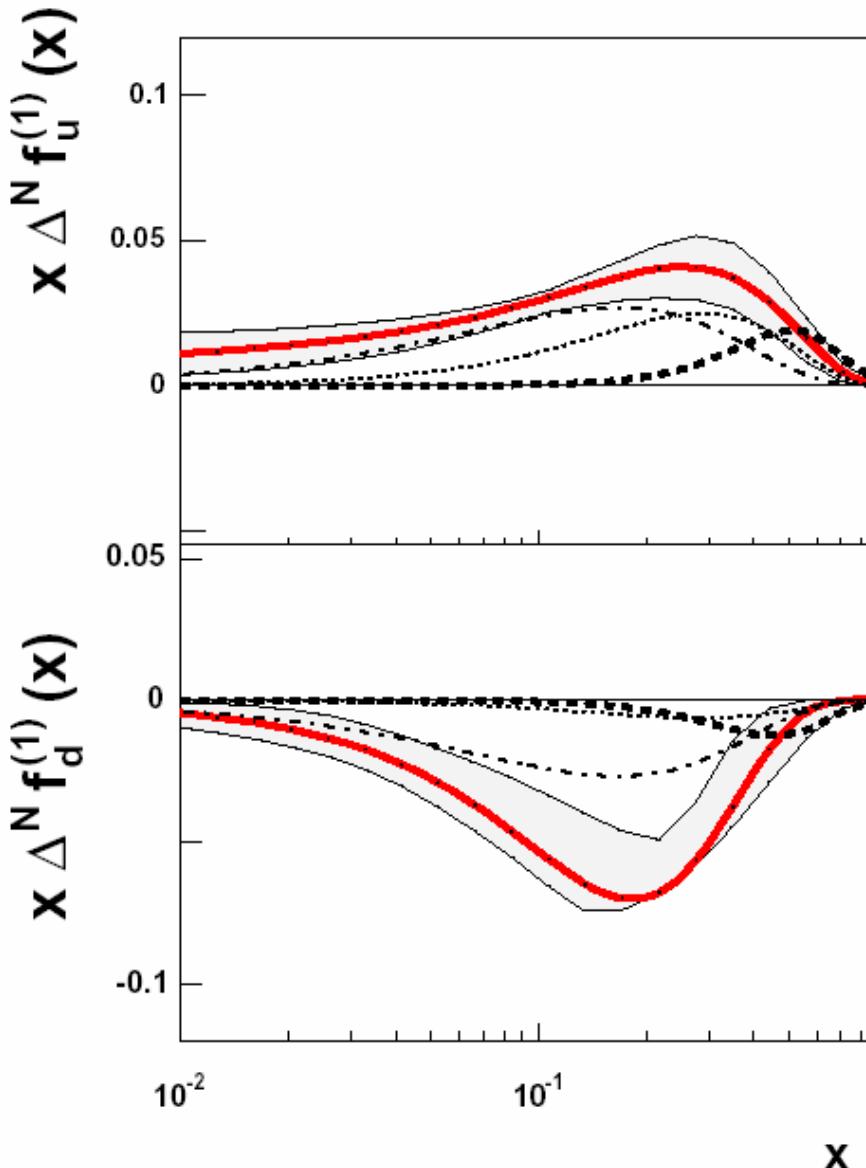


Deuteron target

$$A_{UT}^{\sin(\Phi_h - \Phi_S)} \propto (\Delta^N f_{u/p^\uparrow} + \Delta^N f_{d/p^\uparrow})(4D_u^h + D_d^h)$$



M.A. M. Boglione, U. D'Alesio, A. Kotzinian, F. Murgia, A. Prokudin



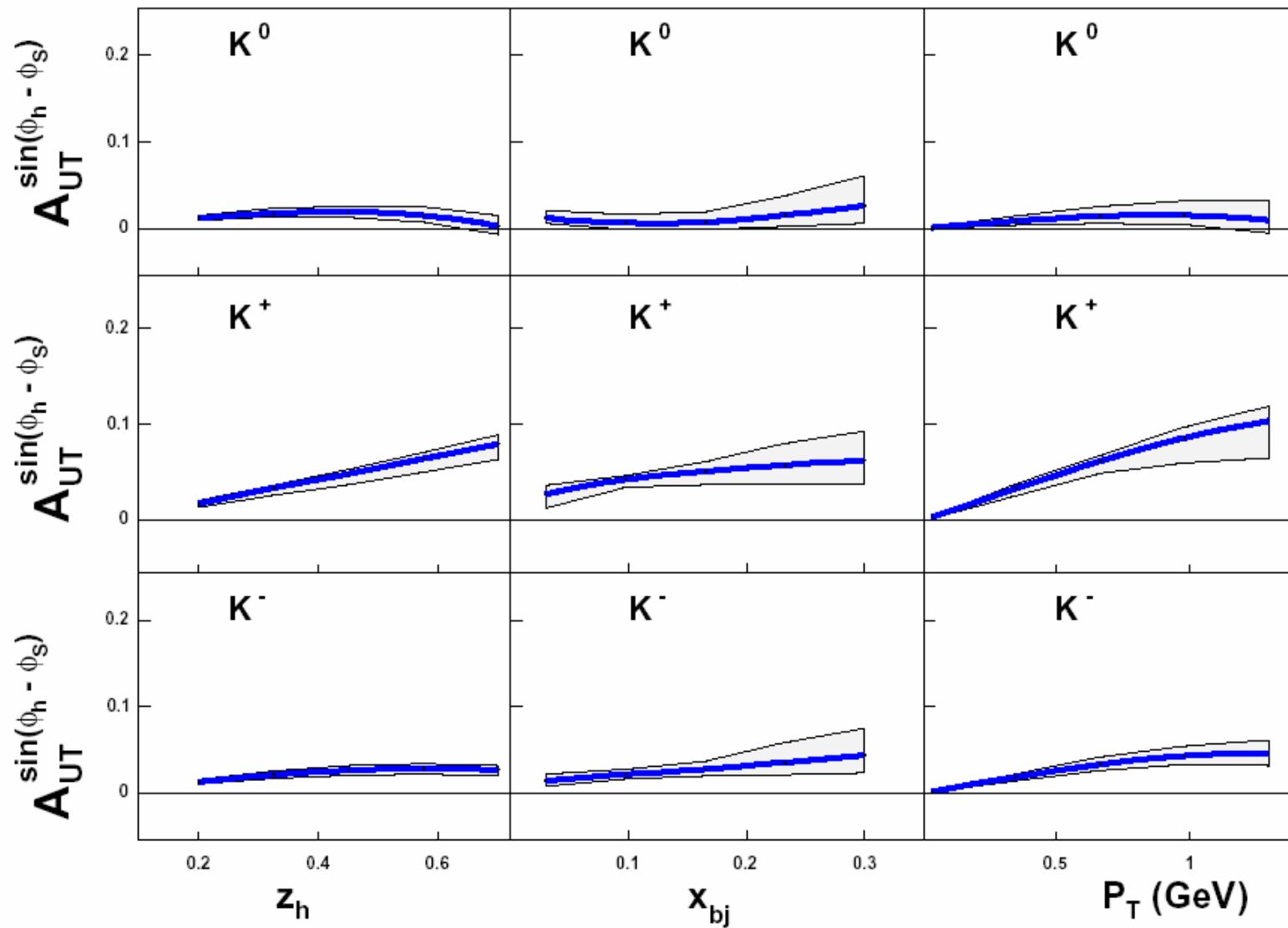
hep-ph/0511017

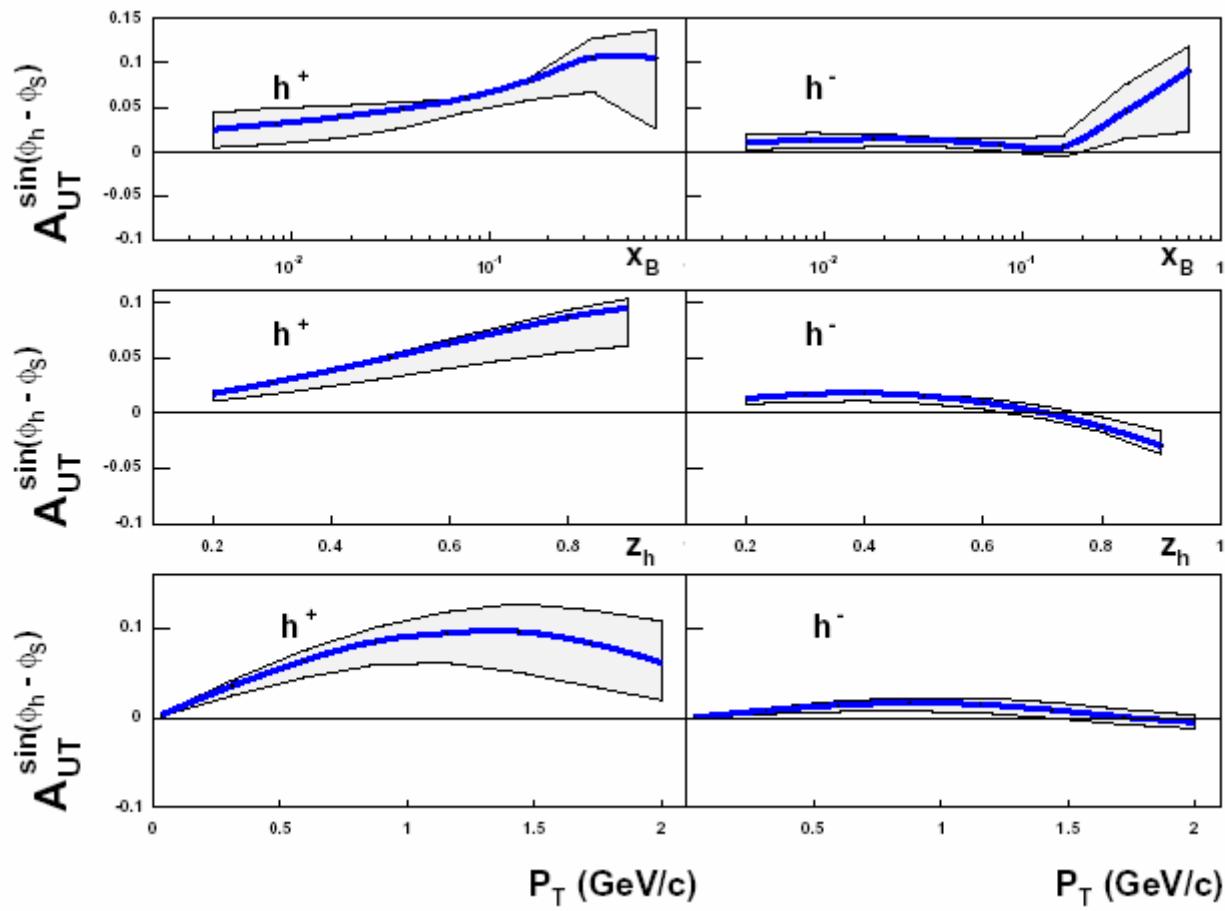
First p_\perp moments of
extracted Sivers
functions, compared
with models

data from HERMES and
COMPASS

$$\Delta^N f_q^{(1)} = -f_{1T}^{\perp(1)q} = \int d^2 k \frac{k_\perp}{4m_p} \Delta^N f_{q/p^\uparrow}(x, k_\perp)$$

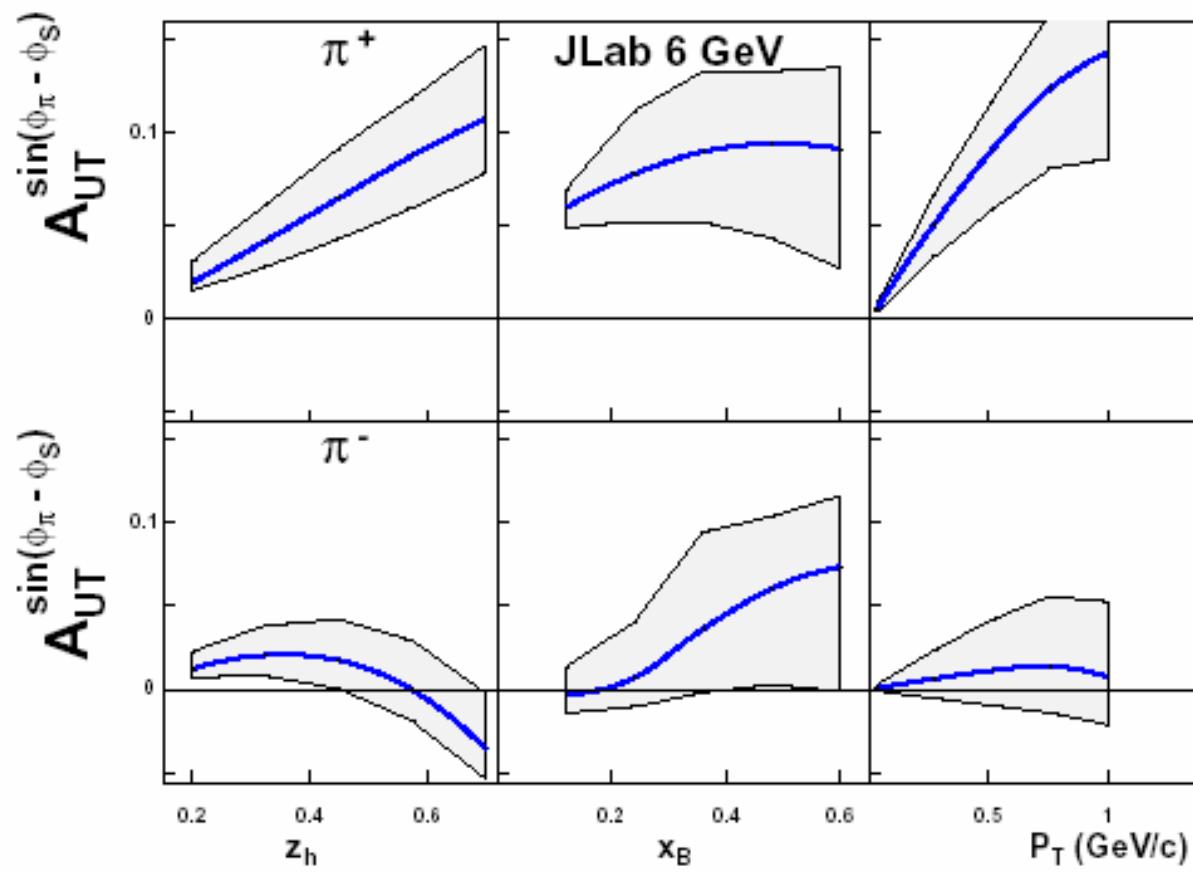
Predictions for K production at HERMES





Predictions for COMPASS,
hydrogen target

$Q^2 > 1 \text{ (GeV/c)}^2$ $W^2 > 25 \text{ GeV}^2$
 $P_T > 0.1 \text{ GeV/c}$ $E_h > 4 \text{ GeV}$
 $0.2 < z_h < 0.9$ $0.1 < y < 0.9$

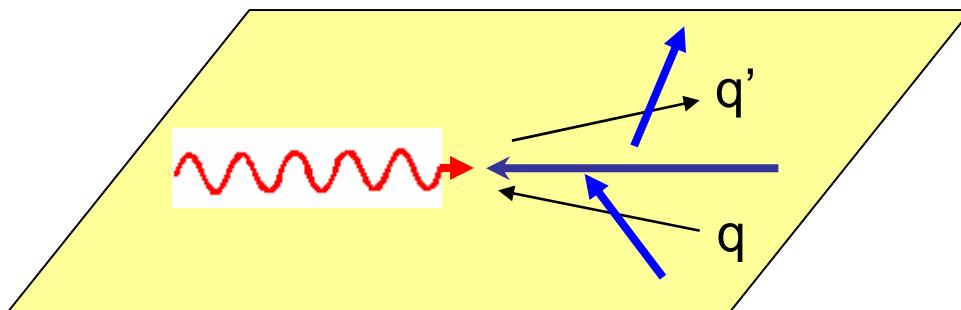


predictions for COMPASS, proton target

Collins mechanism for SSA

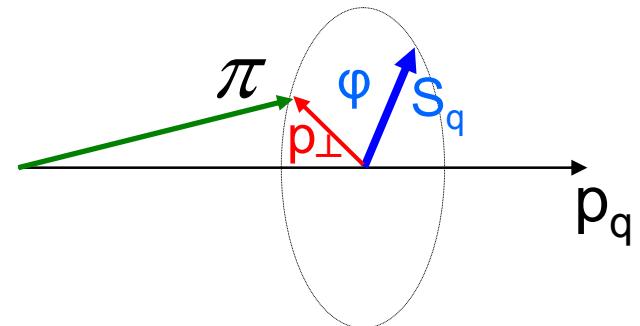
Asymmetry in the fragmentation of a transversely polarized quark

(Fundamental QCD property? D. Sivers)

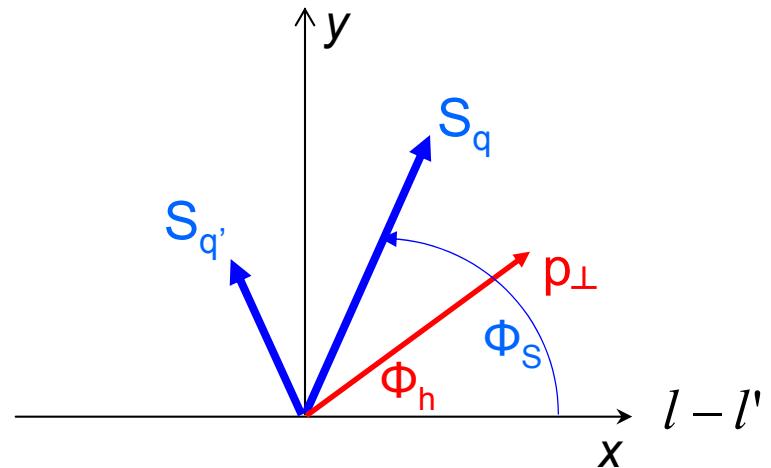


initial q spin is transferred to final q', which fragments

$$\vec{S}_{q'} \cdot (\hat{p}_{q'} \times \hat{p}_\perp) \propto \sin(\Phi_h + \Phi_S)$$



$$D_{h/q^\uparrow}(z, \vec{p}_\perp) = D_{h/q}(z, p_\perp) + \frac{1}{2} \Delta^N D_{h/q^\uparrow}(z, p_\perp) \vec{S}_q \cdot (\hat{p}_q \times \hat{p}_\perp)$$



neglecting intrinsic motion in partonic distributions:

$$A_N^h = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} = \frac{\sum_q \int e_q^2 h_{1q}(x) (1-y)/(xy^2) \Delta^N D_{h/q}^\uparrow(z, p_\perp)}{\sum_q \int e_q^2 f_{q/p}(x) [1+(1-y)^2]/(xy^2) D_{q/p}(z, p_\perp)} \sin(\Phi_h + \Phi_s)$$

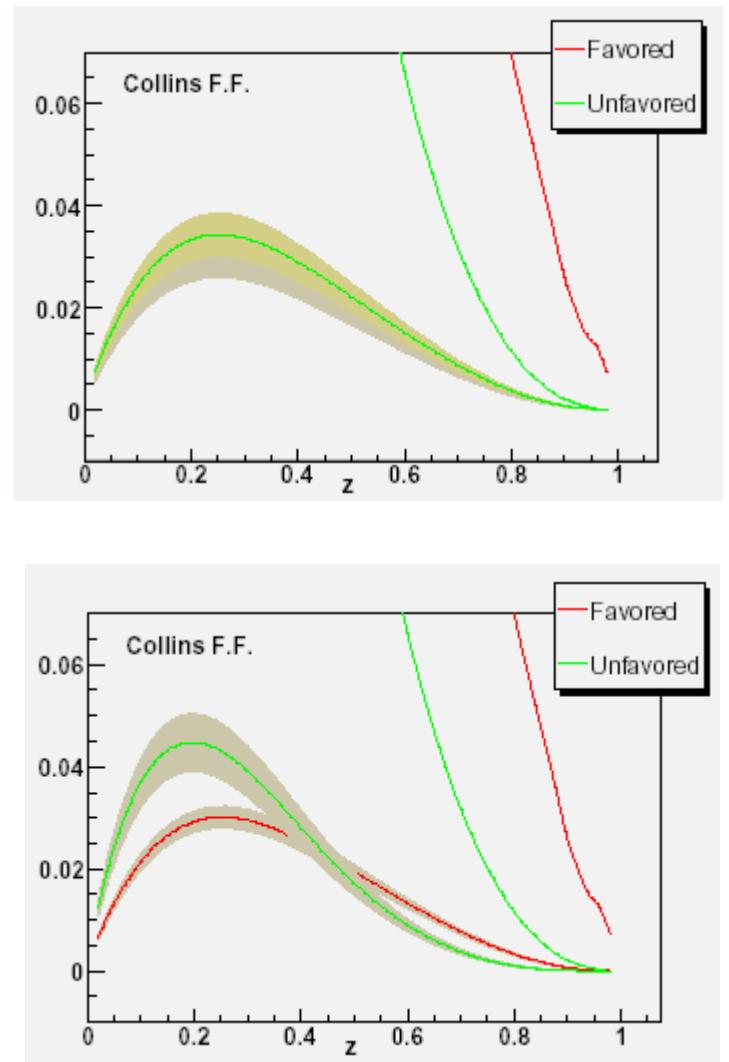
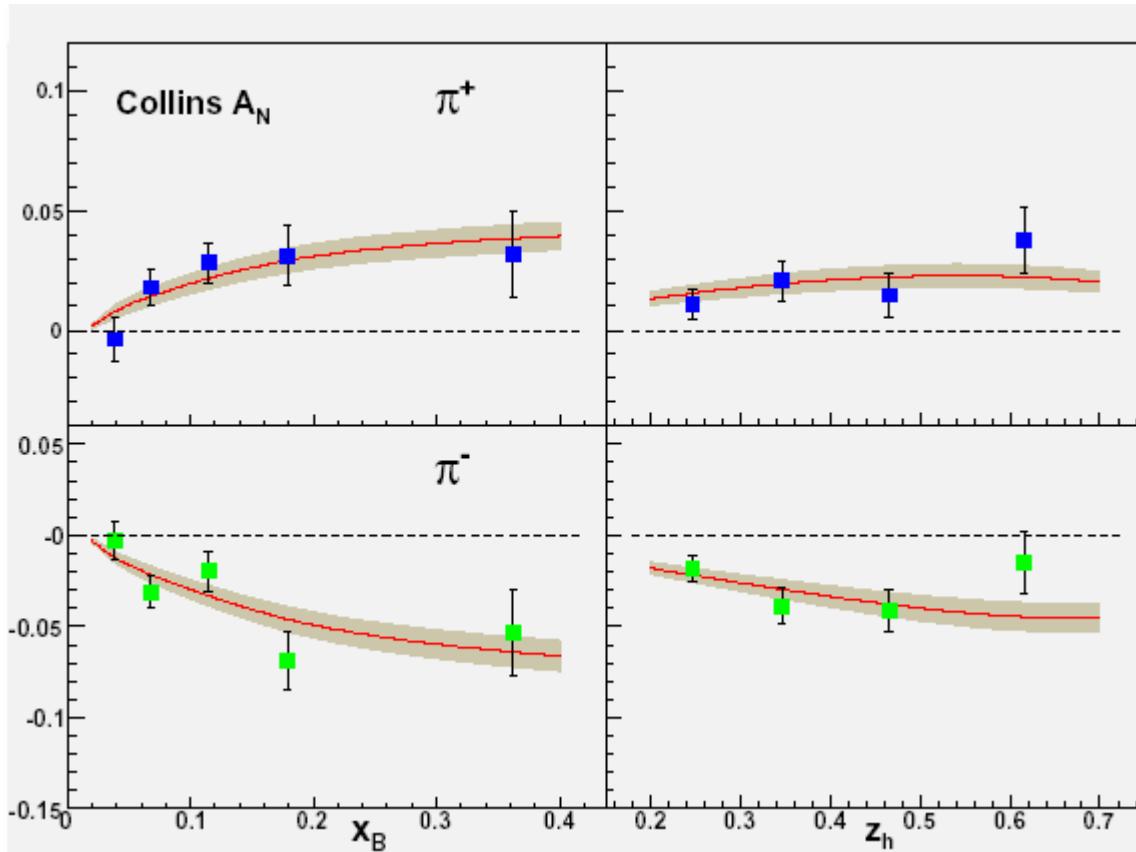
transversity → Collins function

$$A_{UT}^{\sin(\Phi_h + \Phi_s)} \equiv 2 \frac{\int d\Phi_h d\Phi_s [d\sigma^\uparrow - d\sigma^\downarrow] \sin(\Phi_h + \Phi_s)}{\int d\Phi_h d\Phi_s [d\sigma^\uparrow + d\sigma^\downarrow]}$$

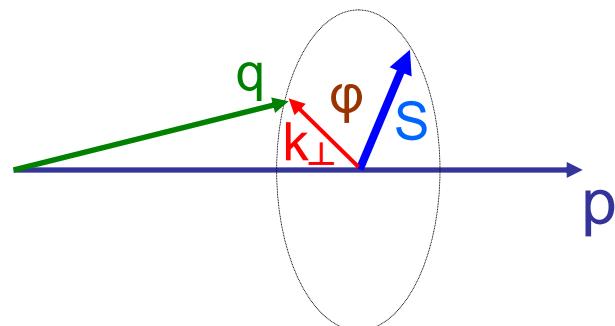
some data available from HERMES, first extraction of Collins functions:
 W. Vogelsang and F. Yuan (assuming Soffer-saturated h_1)

$$(2 | h_1 | \leq \Delta q + q)$$

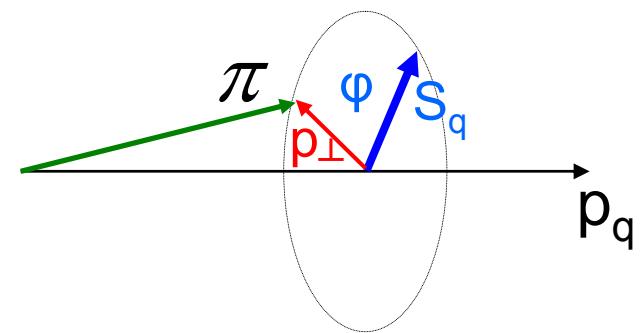
fit to HERMES data on $A_{UT}^{\sin(\Phi_h + \Phi_S)}$



spin- k_\perp correlations



Sivers function



Collins function

$$f_{q/p^\uparrow}(x, \vec{k}_\perp) = f_{q/p}(x, k_\perp) + \frac{1}{2} \Delta^N f_{q/p^\uparrow}(x, k_\perp) \vec{S} \cdot (\hat{p} \times \hat{k}_\perp)$$

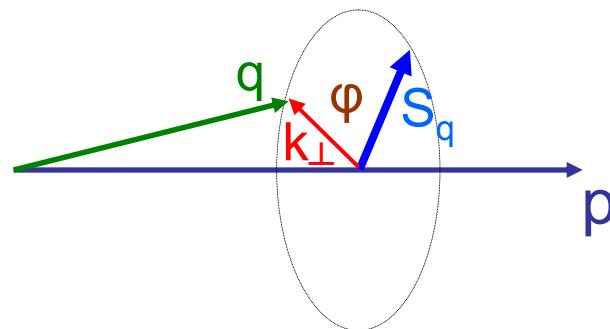
$$D_{h/q^\uparrow}(z, \vec{p}_\perp) = D_{h/q}(z, p_\perp) + \frac{1}{2} \Delta^N D_{h/q^\uparrow}(z, p_\perp) \vec{S}_q \cdot (\hat{p}_q \times \hat{p}_\perp)$$

Amsterdam group notations

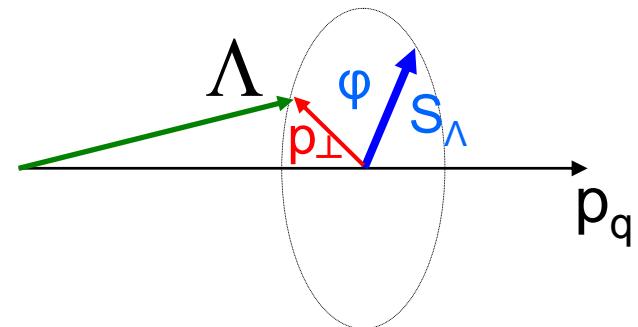
$$\Delta^N f_{q/p^\uparrow} = -\frac{2k_\perp}{M} f_{1T}^{\perp q}$$

$$\Delta^N D_{h/q^\uparrow} = 2 \frac{p_\perp}{z M_h} H_1^{\perp q}$$

spin- k_\perp correlations



Boer-Mulders function



polarizing f.f.

$$f_{q^\uparrow/p}(x, \vec{k}_\perp) = \frac{1}{2} f_{q/p}(x, k_\perp) + \frac{1}{2} \Delta^N f_{q^\uparrow/p}(x, k_\perp) \vec{S}_q \cdot (\hat{p} \times \hat{k}_\perp)$$

$$D_{\Lambda^\uparrow/q}(z, \vec{p}_\perp) = \frac{1}{2} D_{h/q}(z, p_\perp) + \frac{1}{2} \Delta^N D_{\Lambda^\uparrow/q}(z, p_\perp) \vec{S}_\Lambda \cdot (\hat{p}_q \times \hat{p}_\perp)$$

Amsterdam group notations

$$\Delta^N f_{q^\uparrow/p} = -\frac{k_\perp}{M} h_1^{\perp q}$$

$$\Delta^N D_{\Lambda^\uparrow/q} = 2 \frac{p_\perp}{z M_\Lambda} D_{1T}^{\perp q}$$

Hadronic processes: the cross section with intrinsic \mathbf{k}_\perp

$$\frac{E_C d\sigma^{AB \rightarrow CX}}{d^3 p_C} = \sum_{a,b,c,d} \int dx_a dx_b dz d^2 \mathbf{k}_{\perp a} d^2 \mathbf{k}_{\perp b} d^3 \mathbf{k}_{\perp C} \delta(\mathbf{k}_{\perp C} \cdot \hat{\mathbf{p}}_c) \hat{f}_{a/A}(x_a, \mathbf{k}_{\perp a}; Q^2) \hat{f}_{b/B}(x_b, \mathbf{k}_{\perp b}; Q^2) \frac{\hat{s}^2}{\pi x_a x_b z^2 s} J(\mathbf{k}_{\perp C}) \frac{d\hat{\sigma}^{ab \rightarrow cd}}{d\hat{t}}(\hat{s}, \hat{t}, \hat{u}, x_a, x_b) \delta(\hat{s} + \hat{t} + \hat{u}) \hat{D}_{C/c}(z, \mathbf{k}_{\perp C}; Q^2),$$

intrinsic \mathbf{k}_\perp in distribution and fragmentation functions
and in elementary interactions

factorization is assumed, not proven in general; some
progress for Drell-Yan processes, two-jet production, Higgs
production via gluon fusion (Ji, Ma, Yuan; Collins, Metz;
Bacchetta, Bomhof, Mulders, Pijlman)

The polarized cross section with intrinsic \mathbf{k}_\perp

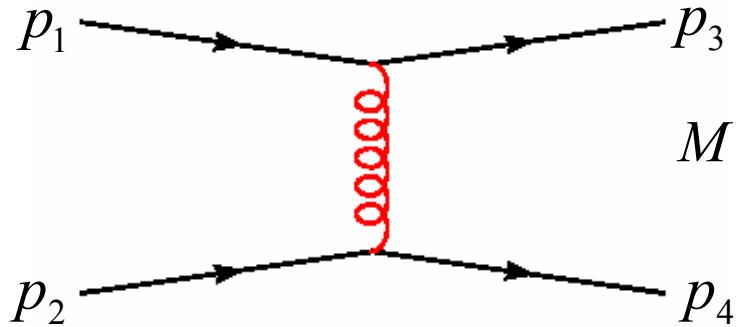
$$\frac{E_C d\sigma^{(A,S_A)+(B,S_B) \rightarrow C+X}}{d^3 p_C} = \sum_{a,b,c,d,\{\lambda\}} \int \frac{dx_a dx_b dz}{16\pi^2 x_a x_b z^2 s} d^2 \mathbf{k}_{\perp a} d^2 \mathbf{k}_{\perp b} d^3 \mathbf{k}_{\perp C} \delta(\mathbf{k}_{\perp C} \cdot \hat{\mathbf{p}}_c) J(\mathbf{k}_{\perp C}) \\ \times \rho_{\lambda_a, \lambda'_a}^{a/A, S_A} \hat{f}_{a/A, S_A}(x_a, \mathbf{k}_{\perp a}) \rho_{\lambda_b, \lambda'_b}^{b/B, S_B} \hat{f}_{b/B, S_B}(x_b, \mathbf{k}_{\perp b}) \quad (1) \\ \times \hat{M}_{\lambda_c, \lambda_d; \lambda_a, \lambda_b} \hat{M}_{\lambda'_c, \lambda'_d; \lambda'_a, \lambda'_b}^* \delta(\hat{s} + \hat{t} + \hat{u}) \hat{D}_{\lambda_c, \lambda'_c}^{\lambda_C, \lambda'_C}(z, \mathbf{k}_{\perp C}),$$

$\rho_{\lambda_a, \lambda'_a}^{a/A, S_A}$ helicity density matrix of parton a inside
polarized hadron A

$\hat{M}_{\lambda_c, \lambda_d; \lambda_a, \lambda_b}$ pQCD helicity amplitudes

$D_{\lambda_c, \lambda'_c}^{\lambda_C, \lambda'_C}$ product of fragmentation amplitudes

Computation of helicity amplitudes



$$M \propto \bar{u}(p_3, \lambda_3) \gamma^\mu u(p_1, \lambda_1) \bar{u}(p_4, \lambda_4) \gamma_\mu u(p_2, \lambda_2)$$

$$p_i = (p_i^0, \mathbf{p}_i)$$

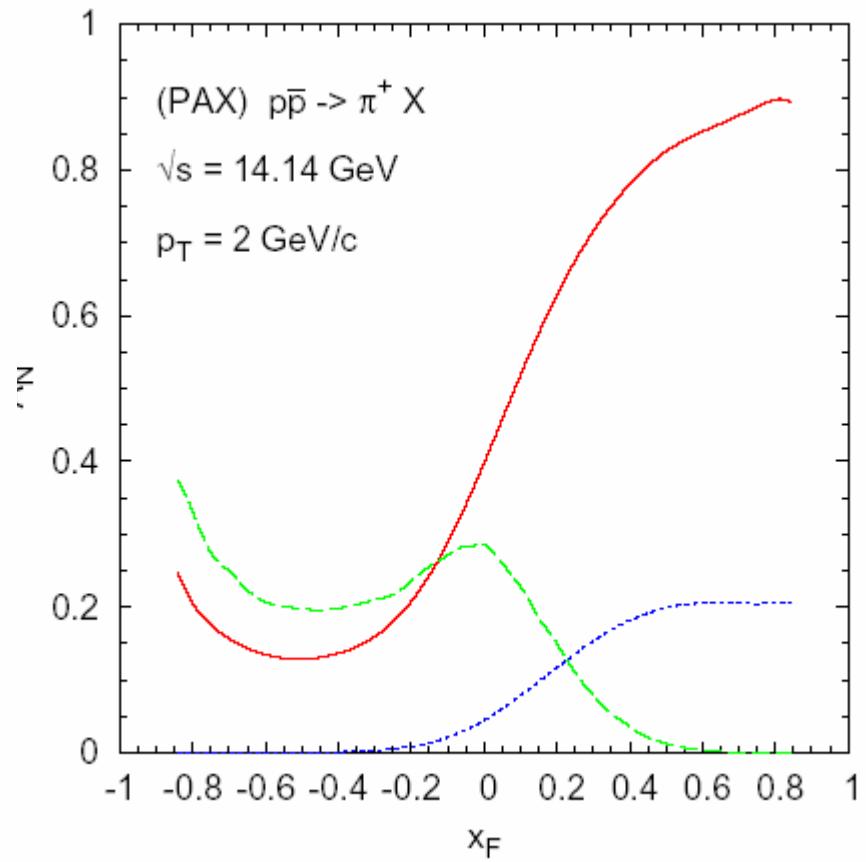
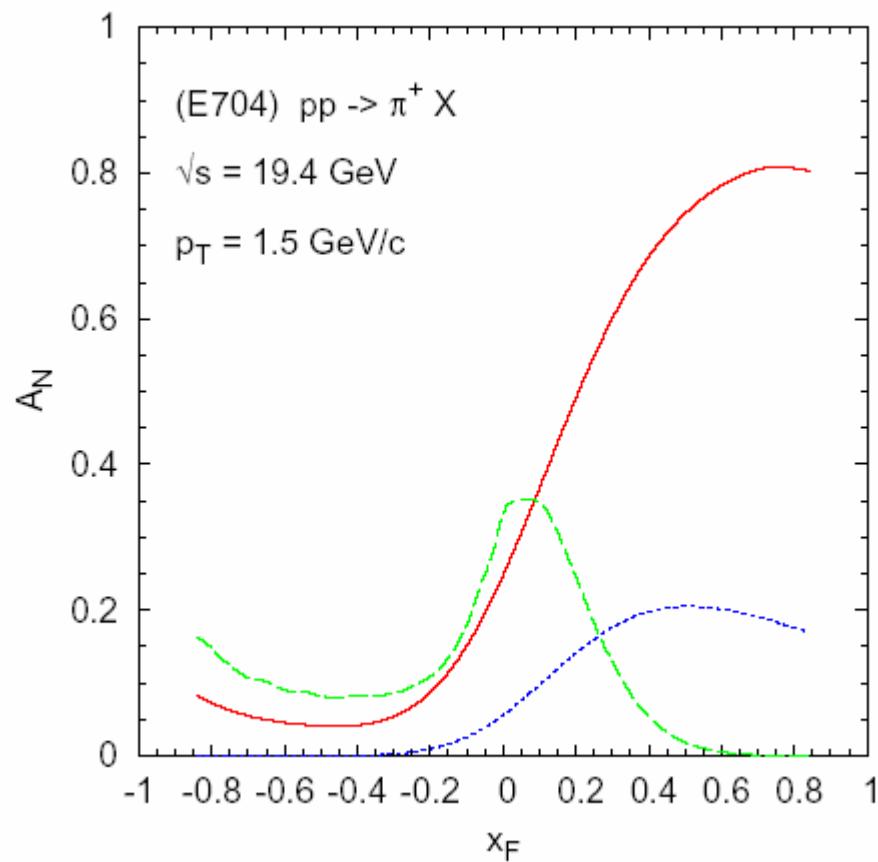
Dirac-Pauli helicity spinors

$$u(p_i, \lambda_i) = \sqrt{p_i^0} \begin{pmatrix} 1 \\ \lambda_i \end{pmatrix} X_{\lambda_i}(\hat{\mathbf{p}}_i) \quad \hat{\mathbf{p}}_i = (\sin \vartheta_i \cos \phi_i, \sin \vartheta_i \sin \phi_i, \cos \vartheta_i)$$

$$X_+(\hat{\mathbf{p}}_i) = \begin{pmatrix} \cos(\vartheta_i/2) e^{-i\phi_i/2} \\ \sin(\vartheta_i/2) e^{i\phi_i/2} \end{pmatrix}$$

$$X_-(\hat{\mathbf{p}}_i) = \begin{pmatrix} -\sin(\vartheta_i/2) e^{-i\phi_i/2} \\ \sin(\vartheta_i/2) e^{i\phi_i/2} \end{pmatrix}$$

if scattering is not planar all Φ_i are different and many phases remain in amplitudes; they strongly suppress the results of integrations over \mathbf{k}_\perp



Maximised (i.e., saturating positivity bounds) contributions to A_N

- quark Sivers contribution
- gluon Sivers contribution
- Collins contribution

SSA in $p\uparrow p \rightarrow \pi X$

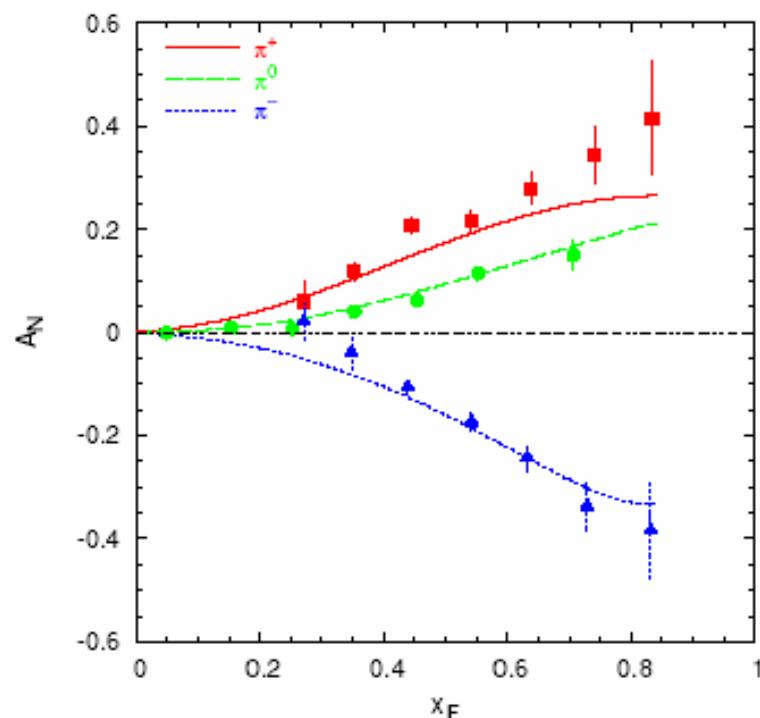
$$d\sigma^\uparrow - d\sigma^\downarrow \simeq \Delta^N f_{a/p} \otimes f_{b/p} \otimes d\hat{\sigma} \otimes D_{\pi/c}$$

“Sivers effect”

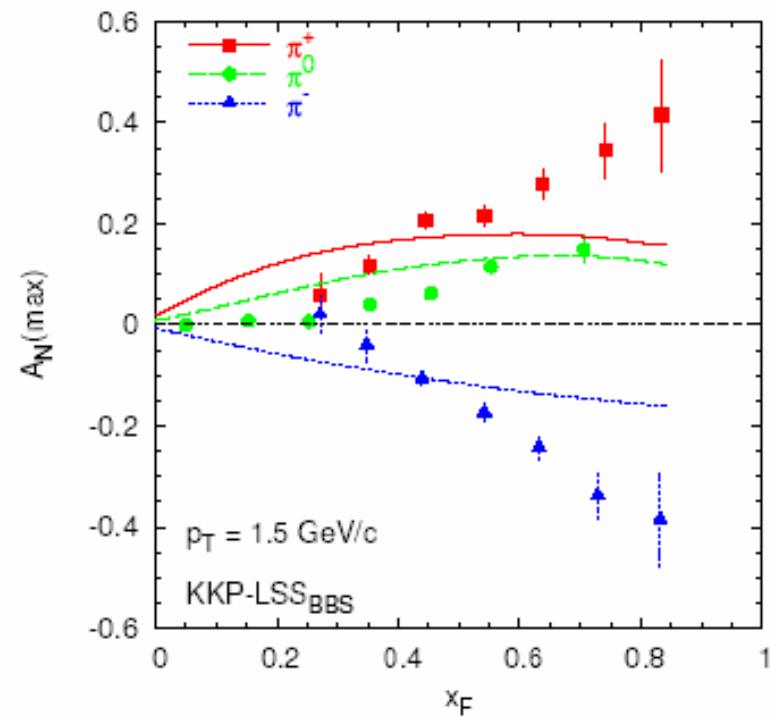
$$+ h_{1a} \otimes f_{b/p} \otimes d\Delta\hat{\sigma} \otimes \Delta^N D_{\pi/c}$$

“Collins effect”

E704 data, $E = 200$ GeV



fit to A_N with Sivers effects alone



maximized value of A_N with Collins effects alone

M.A, M. Boglione, U. D'Alesio, E. Leader, F. Murgia

Conclusions

- Unintegrated (TMD) distribution functions allow a much better description of QCD nucleon structure and hadronic interactions (necessary for correct differential distribution of final state particles, recent paper by Collins, Jung, hep-ph/0508280)
- \mathbf{k}_\perp is crucial to understand observed SSA in SIDIS and pp interactions
- Spin- \mathbf{k}_\perp dependent distribution and fragmentation functions: towards a complete phenomenology of spin asymmetries
- Open issues: factorization, QCD evolution, universality, higher perturbative orders, ...